Unit 3: Algorithms

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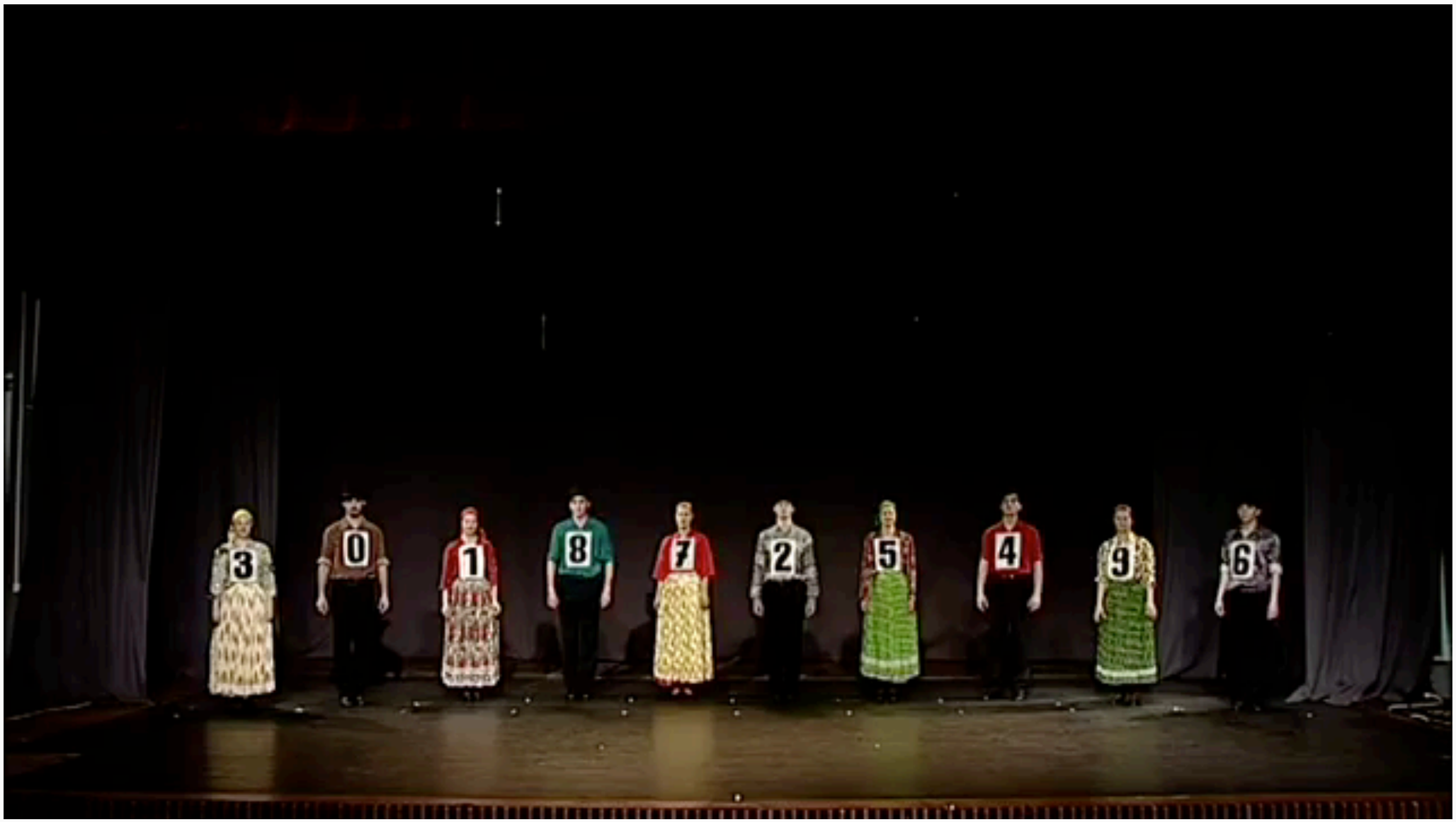
Outline

- Dancing!
- Growth Rates Part 3
- A third problem: *satisfiability*
  - Two algorithms for solving it..
- A fourth problem: *will it halt?*

Dancing Selection Sort



Selection Sort: Growth Rate

- Do this for the first smallest, second smallest, all the way up to Nth smallest:
  - **“Select” the current smallest item in the list.**
  - Put it at the beginning.

takes N steps

doing this N times

So, $N \times N = N^2$

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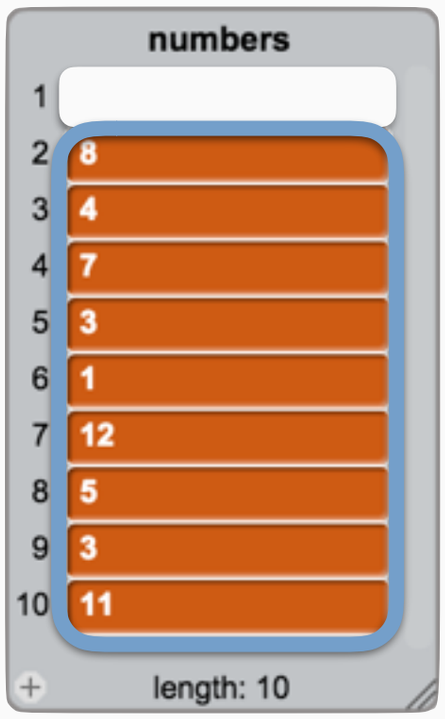

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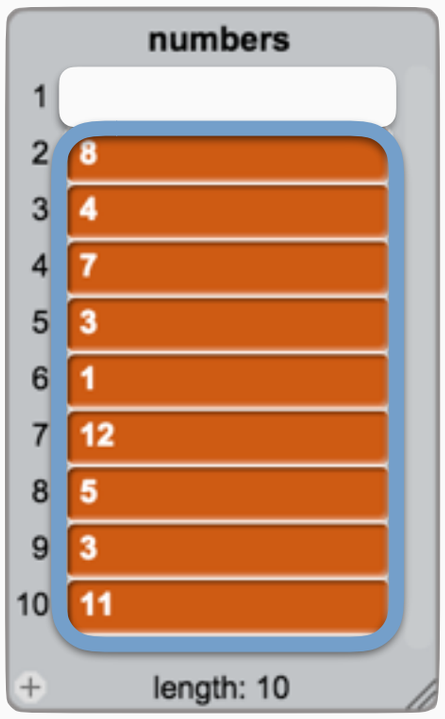

takes N steps

But this is only *N-1*

Q: Does this *really* take *N* Steps?

Selection Sort: Growth Rate

- Do this for the first smallest, **second smallest**, all the way up to Nth smallest:

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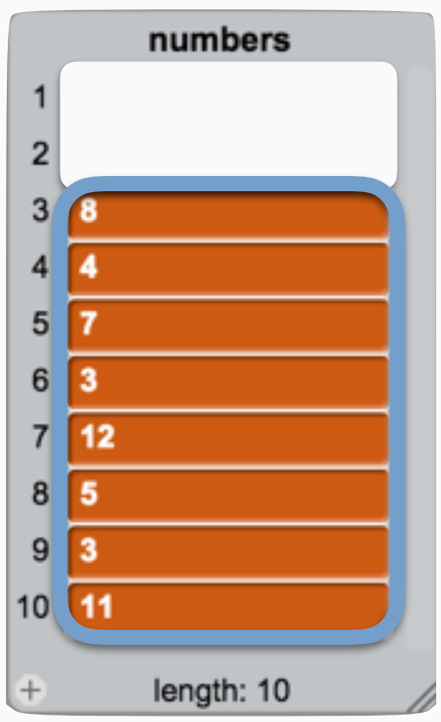
takes *N* steps



But this is only *N-2*

Q: Does this *really* take *N* Steps?

Selection Sort: Growth Rate

- Do this for the first smallest, **second smallest**, all the way up to Nth smallest:

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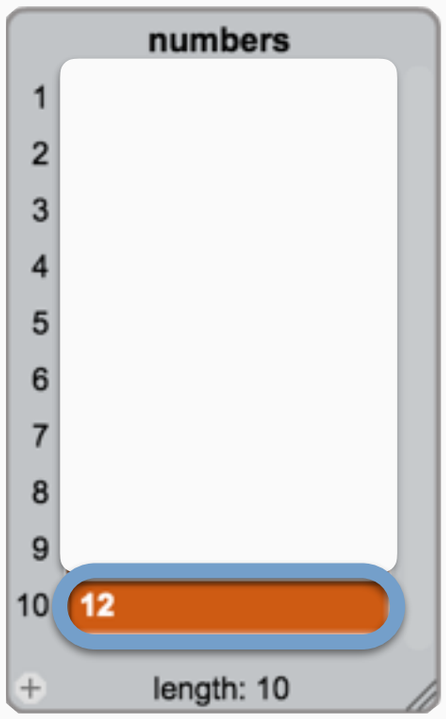
  - Put it at the beginning.

takes *N* steps



But this is only *1* step

Q: Does this *really* take *N* Steps?

Selection Sort: Growth Rate

- Do this for the first smallest, **second smallest**, all the way up to Nth smallest:

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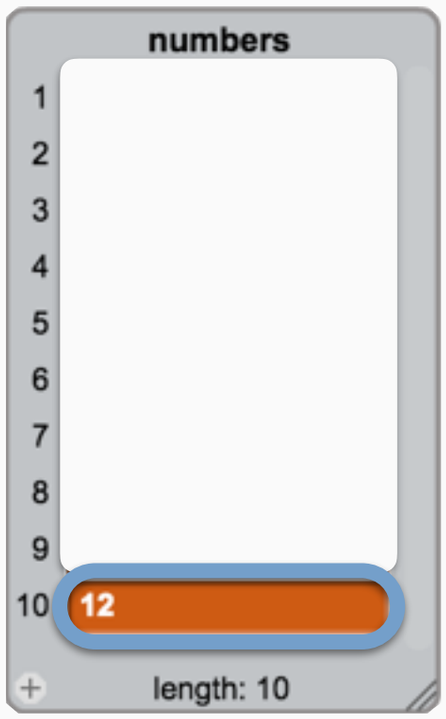
  - Put it at the beginning.



But this is only *1* step

takes N steps

Q: Does this *really* take *N* Steps?

A: First its *N*, then *N - 1*, then *N - 2*...

Selection Sort: Growth Rate

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$$N + (N - 1) + (N - 2) + \ldots 3 + 2 + 1$$

Selection Sort: Growth Rate

A: First its *N*, then *N - 1*, then *N - 2*…

N + (N - 1) + (N - 2) + … 3 + 2 + 1

From Homework: this is just *N(N+1)/2*

Selection Sort: Growth Rate

A: First its *N*, then *N - 1*, then *N - 2…*

N + (N - 1) + (N - 2) + … 3 + 2 + 1

From Homework: this is just *N(N+1)/2* = $(N^2 + N)/2$

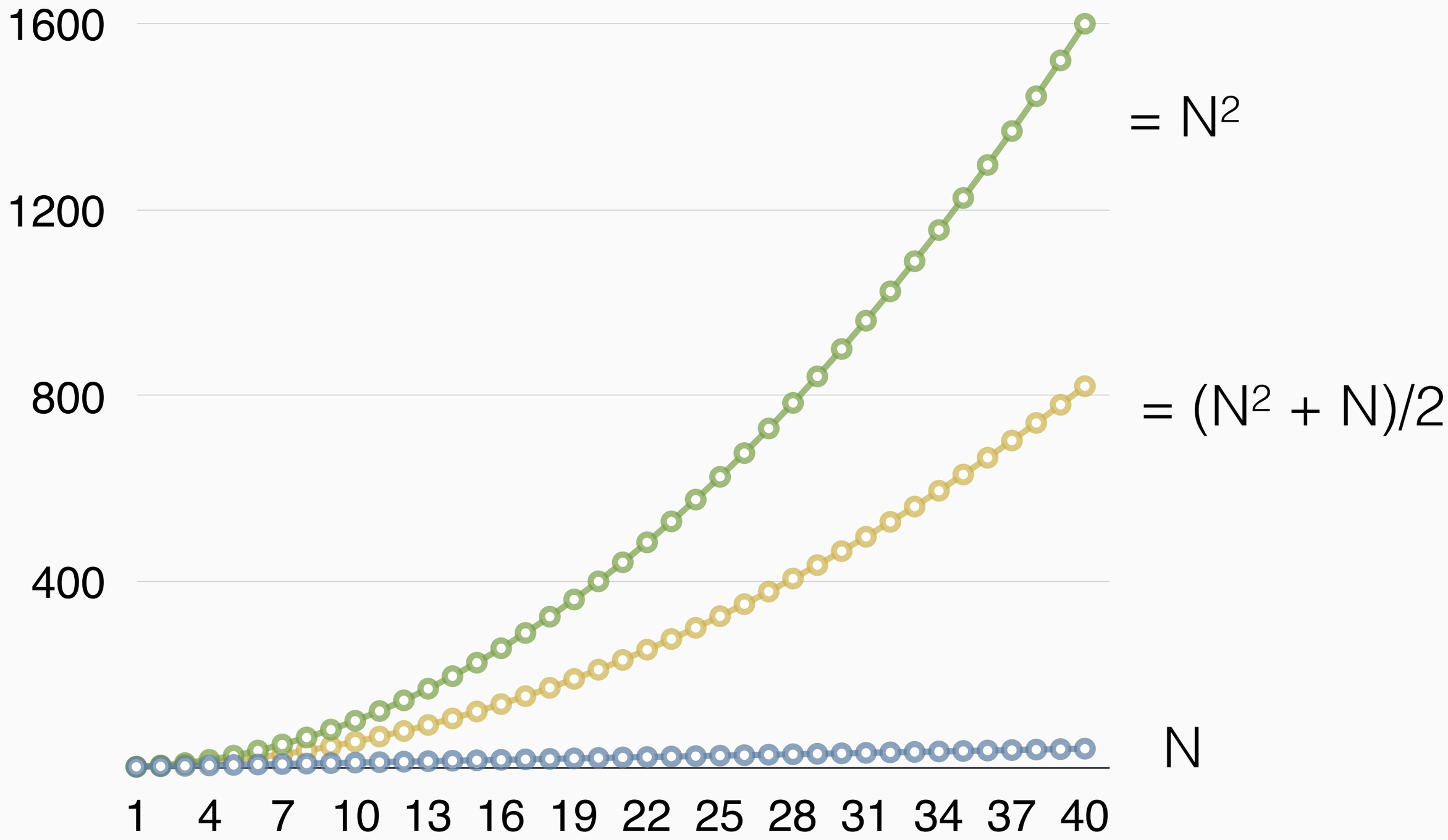
Selection Sort: Growth Rate

A: First its *N*, then *N - 1*, then *N - 2*…

N + (N - 1) + (N - 2) + … 3 + 2 + 1

From Homework: this is just *N(N+1)/2* = (**N²** + N)/2

Selection Sort: Growth Rate



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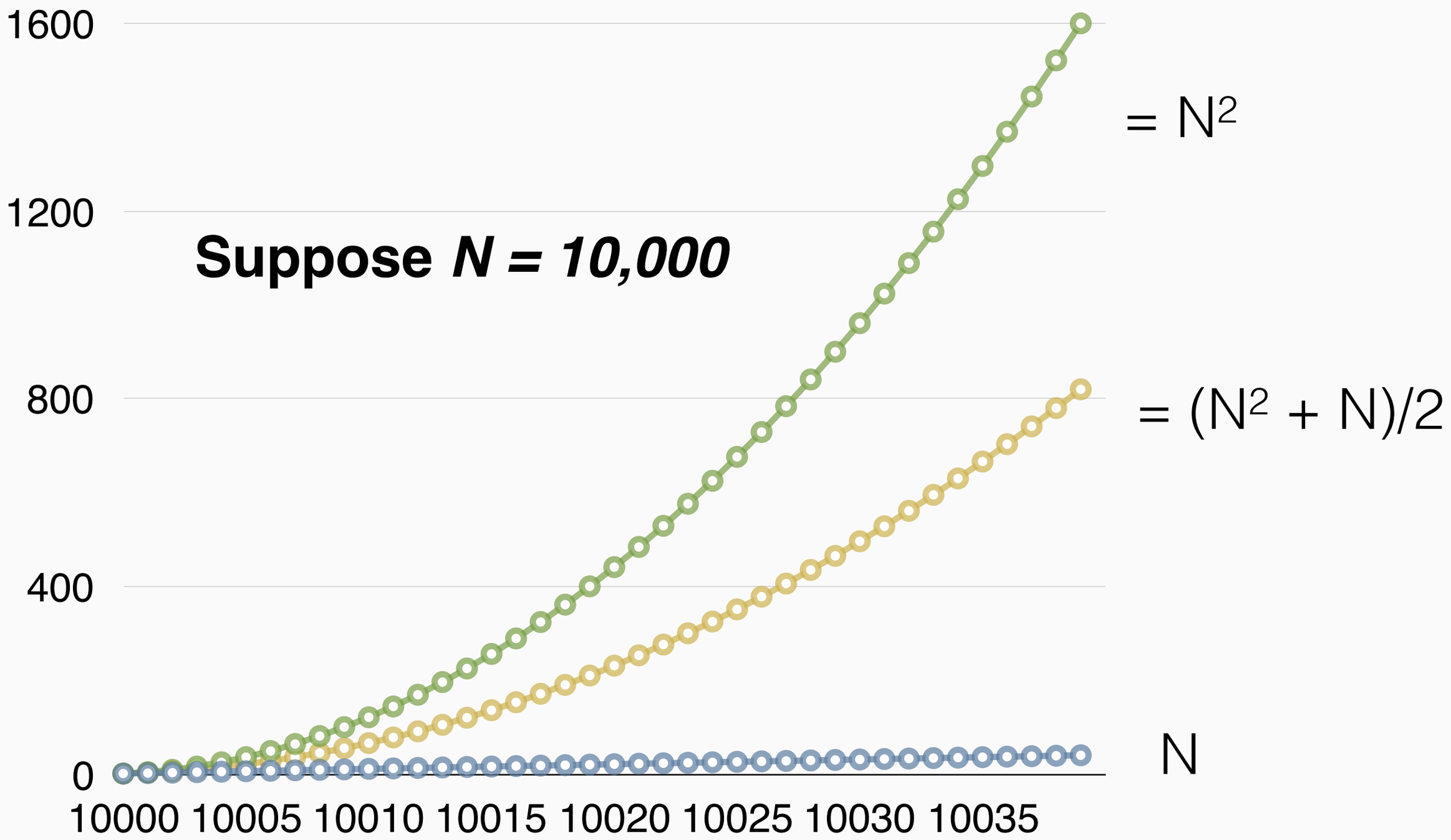

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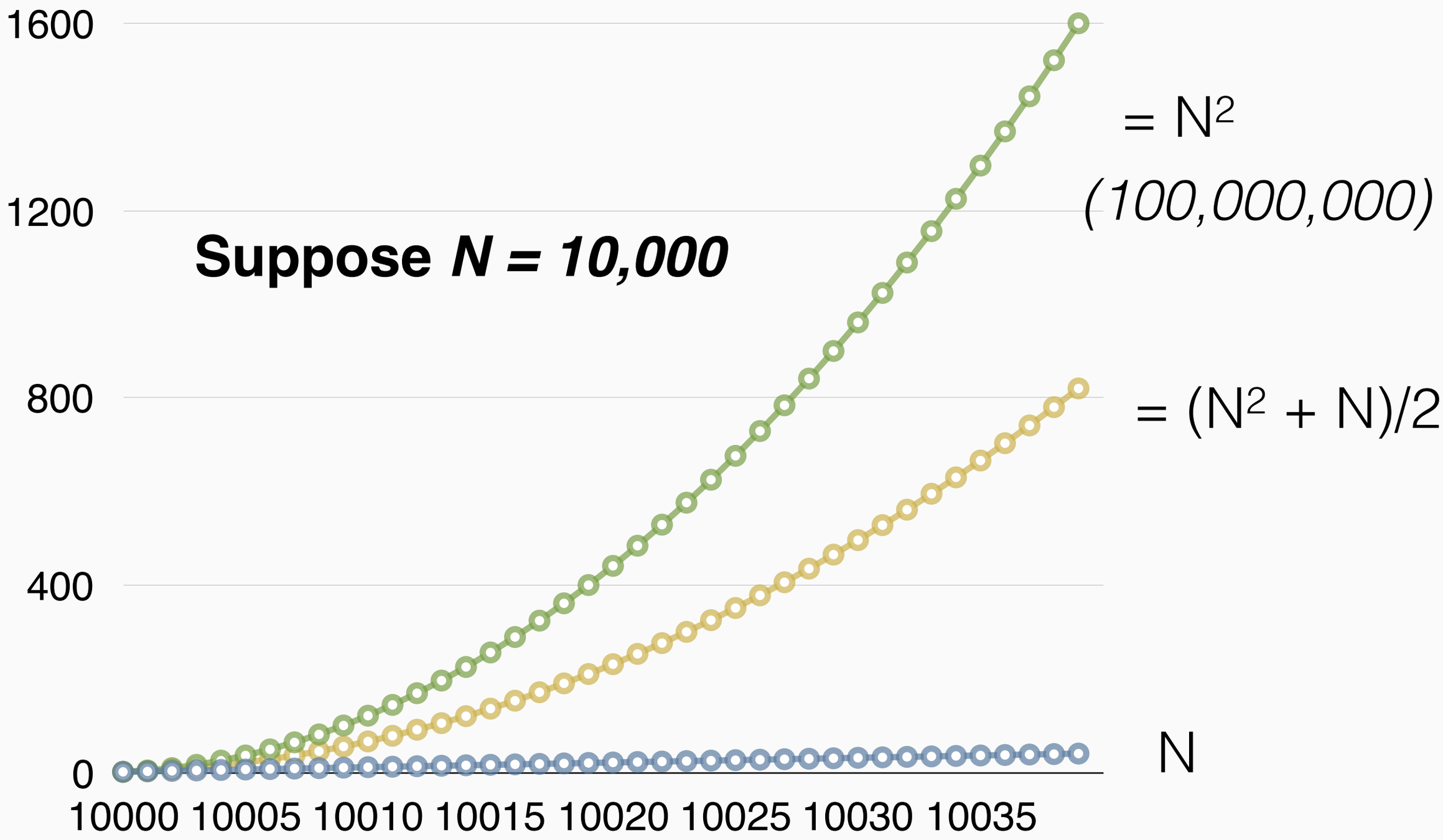

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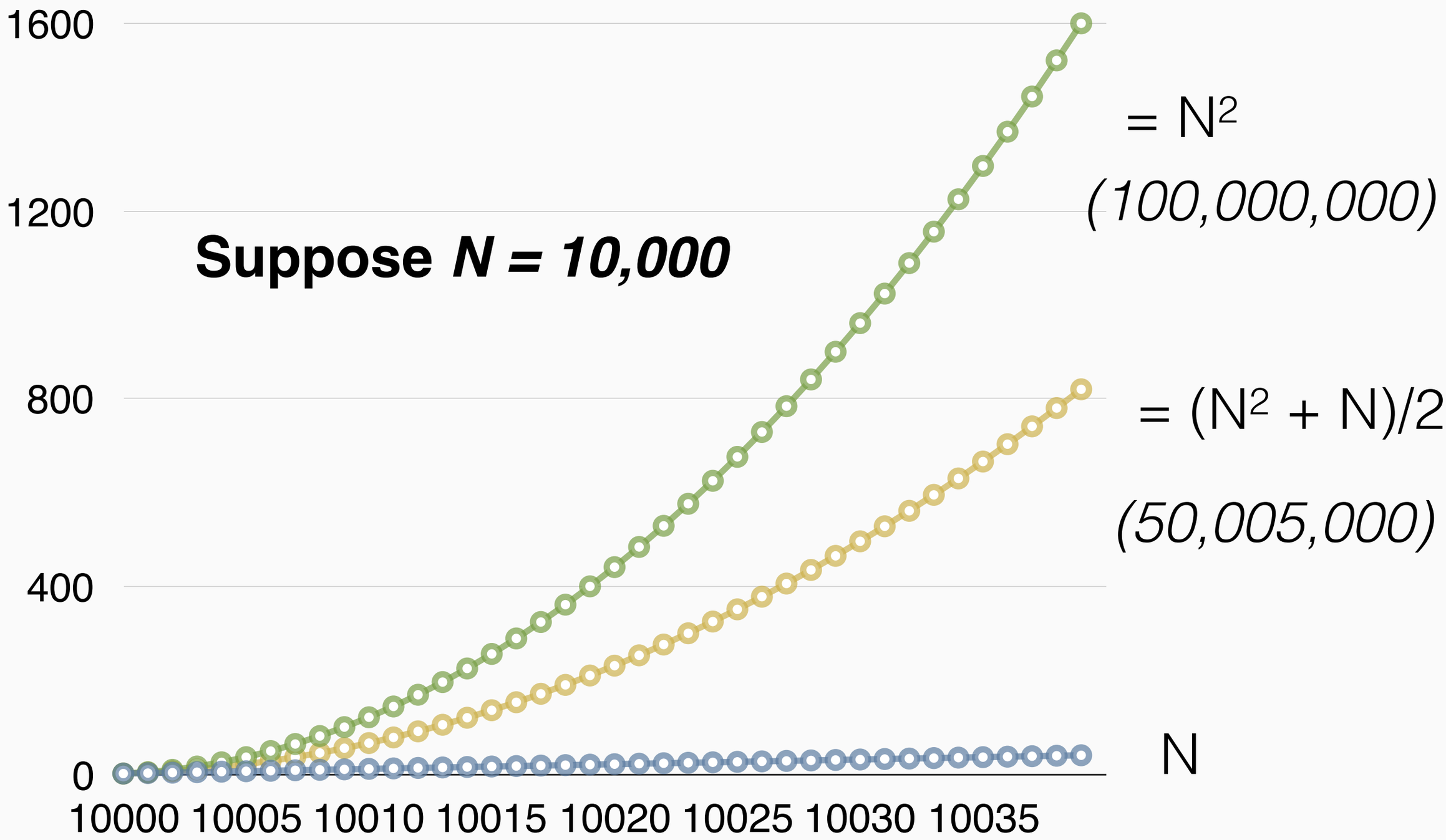

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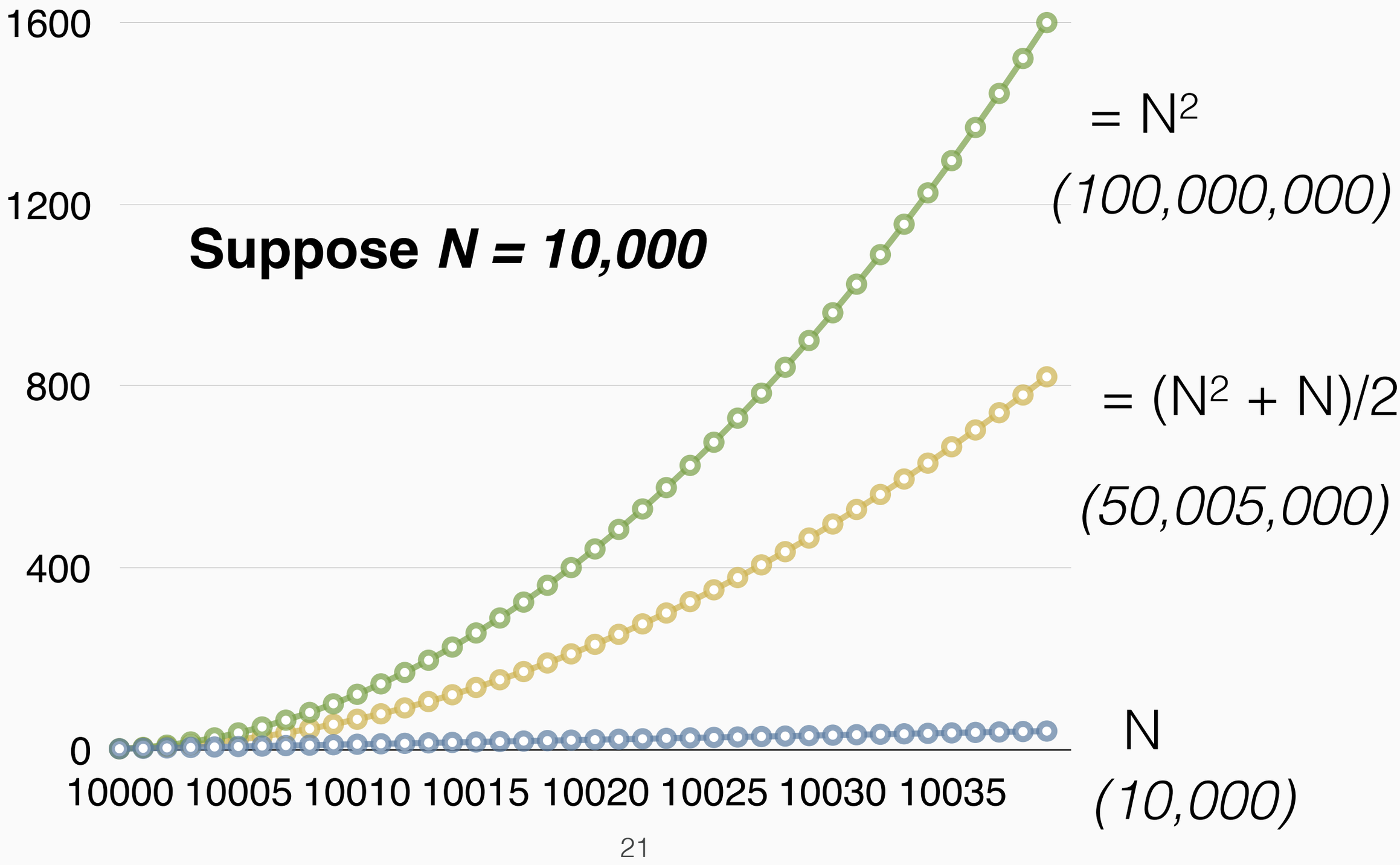

Selection Sort: Growth Rate

Takeaway: we only *really* pay attention to the biggest & most dominant term in the growth rate.

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So: *2N + 1*? Actually just *N*

Selection Sort: Growth Rate

Takeaway: we only *really* pay attention to the biggest & most dominant term in the growth rate.

So: $2N + 1$? Actually just N

So: $3^N + N^2$? Actually just 3^N

Clicker Question!

What is the growth rate of $4N^3 + 2N^2 + N + 3\log_2 N$?

Clicker Question!

[A] $4N^3$

[B] $4N^3 + 2N^2$

[C] N^2

[D] N^3

[D] N

What is the growth rate of $4N^3 + 2N^2 + N + 3log_2N$?

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What is the growth rate of $4N^3 + 2N^2 + N + 3log_2N$?

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[A] $4N^3$ [B] $4N^3 + 2N^2$ [C] N^2

[D] N^3 [D] N

Since N^3 is the most dominant term… We don't care about constants

What is the growth rate of $4N^3 + 2N^2 + N + 3log_2N$?

A Third Problem:
Can this logical formula be true?

- *INPUT:*
  - *A* logical formula
- *OUTPUT:*
  - True if there is some way we can make the formula true
  - False if there is no way we can make the formula true

A Third Problem: *Satisfiability*

- *INPUT:*
  - A boolean sentence.
- *OUTPUT:*
  - True if there is some way we can make the formula true
  - False if there is no way we can make the formula true

Satisfiability

I give you: *AND(P,Q)*

Q: Can this be true?

(our tools are how we set the truth value of *P, Q*)

Satisfiability

I give you: *AND(P,Q)*

Q: Can this be true?

A: Sure! *P = True, Q = True*

(our tools are how we set the truth value of *P, Q*)

Satisfiability

- Instead of our growth rate capturing the length of the list, now it measures the *number of atomic sentences, e.g. P, Q, R, etc.*

- So, how hard is this problem?

- Let's try a harder one…

Clicker Question!

Q: Can the following logical formula be made True?

NOT(OR(AND(OR(P,Q),NOT(R)),R))

or if you'd prefer the other way of writing things…

not(((P or Q) and not(R)) or R)

Clicker Question!

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or if you'd prefer the other way of writing things…

not(((P or Q) and not(R)) or R)

[A] Yes, I'm very confident! [B] No, I'm very confident!

[C] I'm guessing randomly!

Clicker Answer!

Q: Can the following logical formula be made True?

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or if you'd prefer the other way of writing things…

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A: Yes! *P = False, Q = False, R = False*

Algorithm 1: Satisfiability

Random Checker

1. Pick a random assignment for each atomic sentence's truth value.

2. Ask, is the sentence True?

3. If it is, report True.

4. If not, repeat from step 1.

Satisfiability: Random Checker

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(P and Q) or not(P)

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The usual questions… Q: It is *correct?* A: No!

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The usual questions… Q: What's the growth rate?

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4. If not, repeat from step 1.

The usual questions…

Q: What's the growth rate?

A: Actually hard to say…

Algorithm 2: Satisfiability

Build The Truth Table! (Brute Force)

1. *Build the truth table for the logical formula.*
2. *Check to see if it has any row that is True.*
3. *If it does, report True.*
4. *If not, report False.*

Satisfiability: Build the Table!

(P and Q) or not(P)

Build the Table!

1. Build the Truth Table
2. Check to see if any row is True.
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4. Otherwise, report False.

Clicker Question!

(((P and Q) or not(R)) or S)

Q: How many rows will there be in the truth table?

[A] 8 [B] 12 [C] 14 [D] 16 [E] 18

Clicker Question!

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Hint 1: We need each combination of P, Q, and R being True/False…

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Hint 3: We have three atomic sentences, i.e. three “bits”

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Clicker Answer!

P	*Q*	*R*	*S*
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

Q: How many rows will there be in the truth table?

Clicker Answer!

Q: How many rows will there be in the truth table?

Can think about it as an N bit binary number!

P	*Q*	*R*	*S*
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

Clicker Answer!

Q: How many rows will there be in the truth table?

Can think about it as an N bit binary number!

P	*Q*	*R*	*S*
1	1	1	1
1	1	1	0
1	1	0	0
1	1	0	0
1	0	1	0
1	0	1	0
1	0	0	1
1	0	0	0
0	1	1	1
0	1	1	0
0	1	0	1
0	1	0	0
0	0	1	1
0	0	1	0
0	0	0	1
0	0	0	0

Clicker Answer!

Q: How many rows will there be in the truth table?

Can think about it as an N bit binary number!

More generally: for a sentence of N variables, there will be 2^N combinations.

P	*Q*	*R*	*S*
1	1	1	1
1	1	1	0
1	1	0	0
1	1	0	0
1	0	1	0
1	0	1	0
1	0	0	1
1	0	0	0
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0	1	1	0
0	1	0	1
0	1	0	0
0	0	1	1
0	0	1	0
0	0	0	1
0	0	0	0

Satisfiability: Build the Table!

(P and Q) or not(P)

Build the Table!

1. Build the Truth Table
2. Check to see if any row is True.
3. If so, report True.
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Satisfiability: Build the Table!

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(P and Q) or not(P)

P	*Q*	*R*	*(P and Q)*	*or*	*not(P)*
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
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P	*Q*	*R*	*(P and Q)*	*or*	*not(P)*
T	T	T	T	T	F
T	T	F	T	T	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	F	T	T
F	T	F	F	T	T
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T	T	F	T	T	F
T	F	T	F	F	F
T	F	F	F	F	F
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True!

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T	T	F	T	T	F
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T	F	F	F	F	F
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T
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Satisfiability: Build the Table!

- Q1: Will it halt?
- Q2: Is it correct?
- Q3: What's the growth rate?

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Satisfiability: Build the Table!

- Q1: Will it halt?

  A: Yes! Build a truth table…

- Q2: Is it correct?

  A: Yes! Truth table tells all…

- Q3: What's the growth rate?

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- Q3: What's the growth rate?

  A: Clicker question!

Build the Table!

1. Build the Truth Table
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Clicker Question!

[A] 2^N [B] N [C] N^2 [D] N*log(N)

N = number of atomic sentences in the formula (e.g. *P, Q*)

Q3: What's the growth rate of the "build the table" algorithm?

Clicker Answer!

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"Primitive operation" here is building a truth table row

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"Primitive operation" here is building a truth table row

[A] 2^N [B] N [C] N^2 [D] N*log(N)

We just said there were 2^N rows for an *N* variable formula

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Q3: What's the growth rate of the "build the table" algorithm?

Satisfiability: Build the Table!

- Q1: Will it halt?

  A: Yes! Build a truth table…

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  A: 2^N

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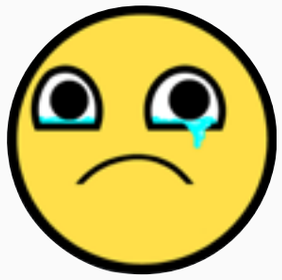
- Q3: What's the growth rate?

  A: 2^N



If $N = 100$, this is already basically unsolvable…

Build the Table!

1. Build the Truth Table
2. Check to see if any row is True.
3. If so, report True.
4. Otherwise, report False.

Satisfiability: Build the Table!

- Q1: Will it halt?

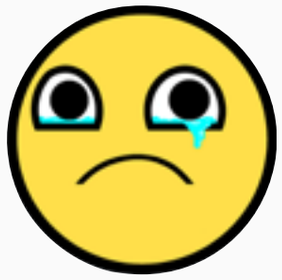
  A: Yes! Build a truth table…

- Q2: Is it correct?

  A: Yes! Truth table tells all…

- Q3: What's the growth rate?

  A: 2^N



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Can We Do Better?

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Build the Table!

1. Build the Truth Table
2. Check to see if any row is True.
3. If so, report True.
4. Otherwise, report False.

Clicker Question!

[A] Yes!

[B] No!

Q: Is there a correct algorithm to solve the Satisfiability problem with a growth rate faster than 2^N?

Clicker Answer!

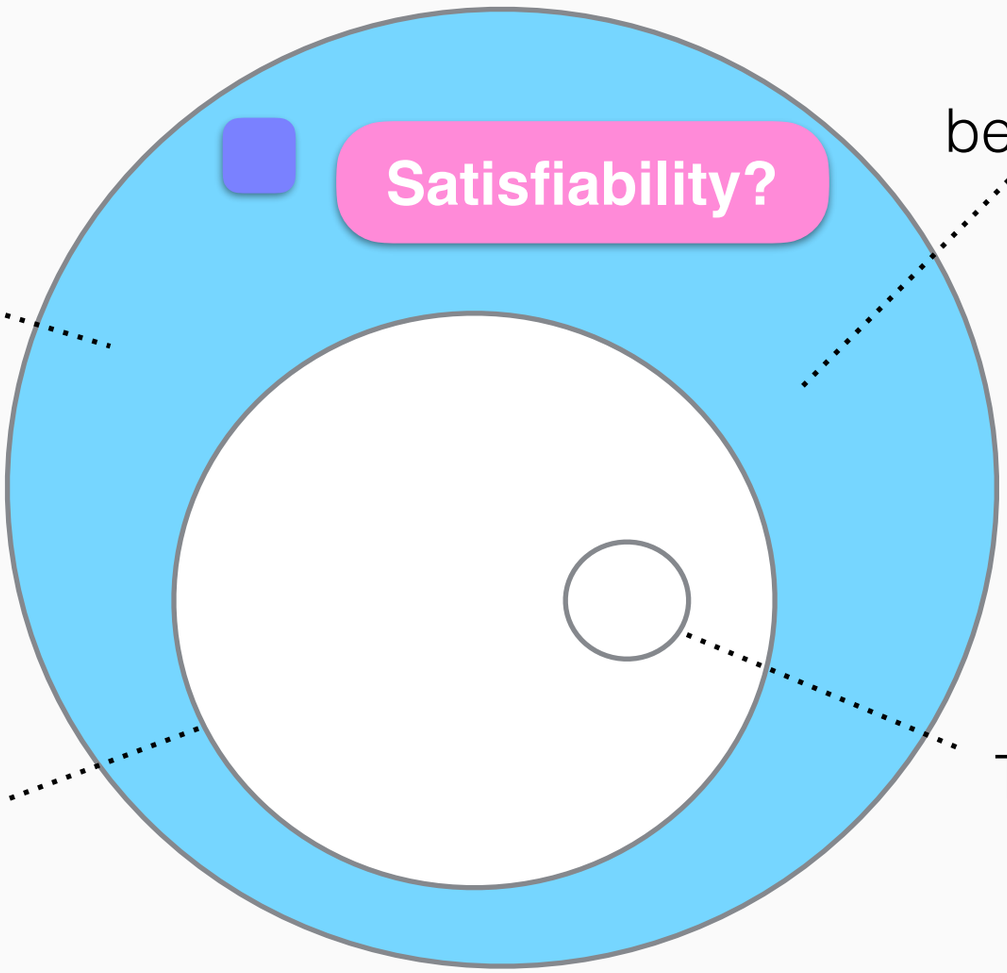
[A] Yes! [B] No!

A: We don't know!

Q: Is there a correct algorithm to solve the Satisfiability problem with a growth rate faster than 2^N?

Computability

Q: What, if anything, is out here?



Things that can be computed, period.

Things a regular computer can compute before the sun goes supernova

Things a domino computer could compute before the sun goes supernova

Computability

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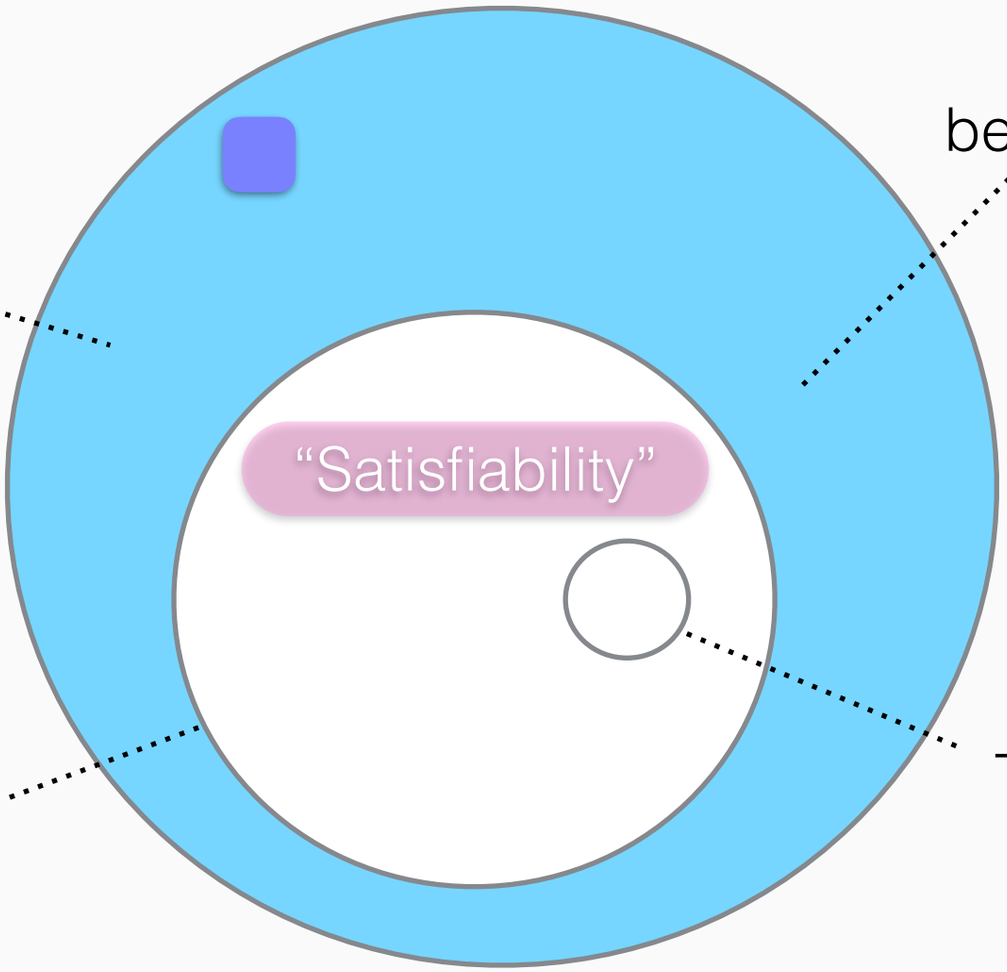

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A Final Problem: *Does it Halt?*

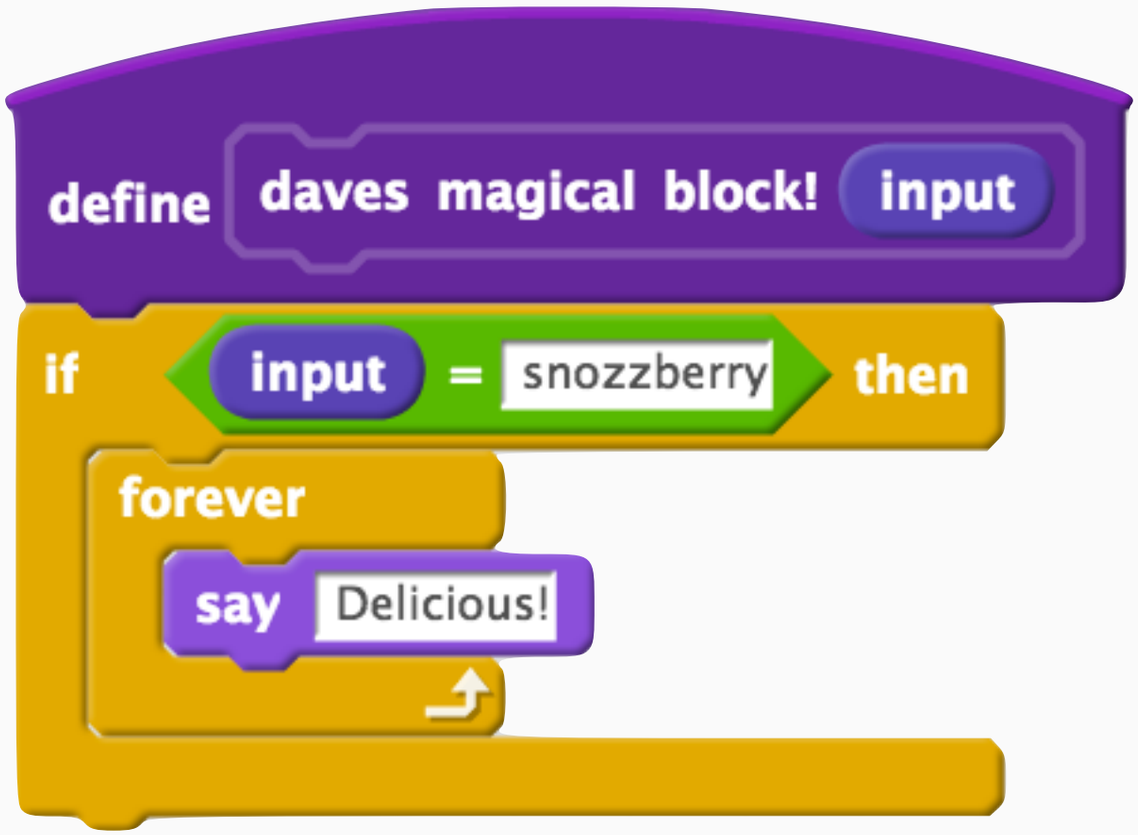
- *INPUT:*
  - A description of an algorithm, input to the algorithm.
- *OUTPUT:*
  - True if the algorithm will halt when run on that input.
  - False if the algorithm does not halt with that input.

Example: *Does it Halt?*

Example: *Does it Halt?*

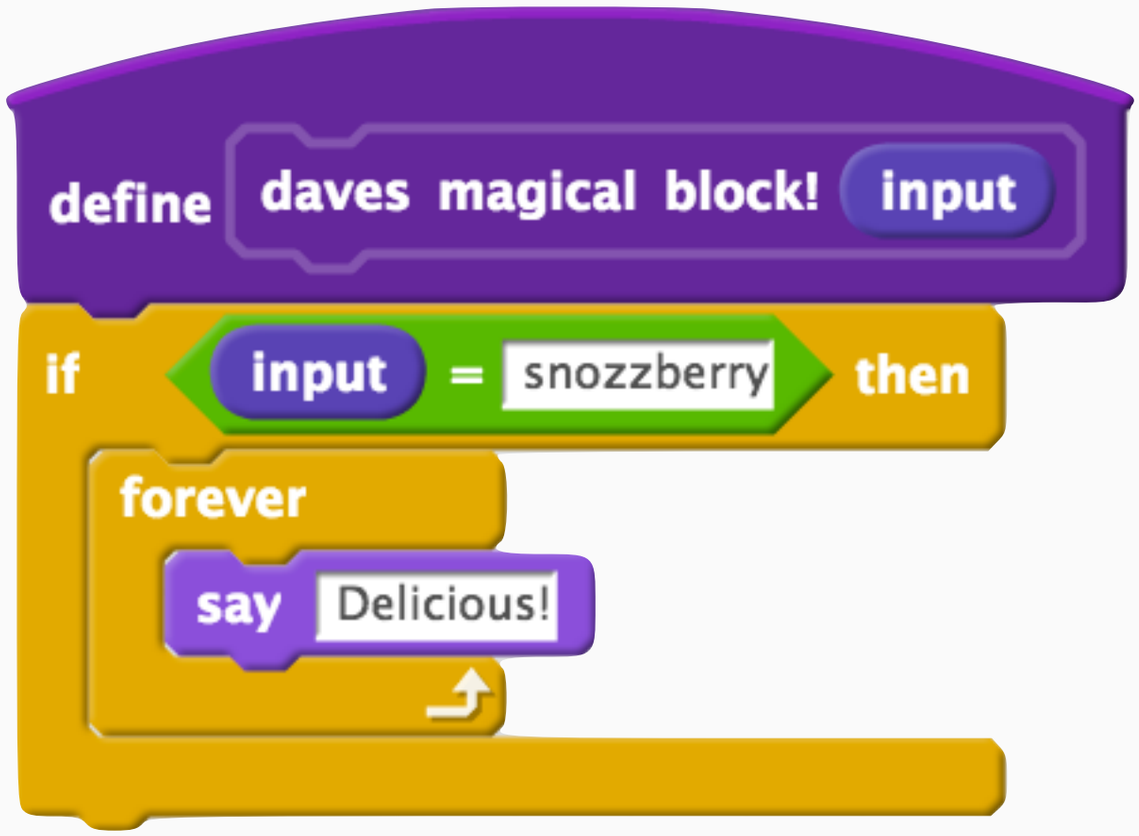
A: No!

Example: *Does it Halt?*



Example: *Does it Halt?*

A: No!



Algorithm: Does *it Halt?*

Loop Checker!

1. Does the given algorithm use any infinite loops?

2. If so, report False.

3. If not, report True.

Algorithm: Does *it Halt?*

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  A: *N*, have to check each step of the given algorithm.

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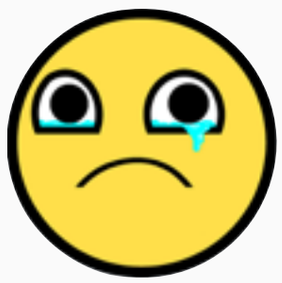
  A: No! 

- Q3: What's the growth rate?

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Q: Can We Do Better?

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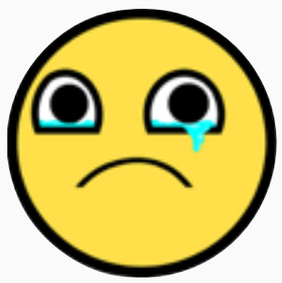
  A: No! 

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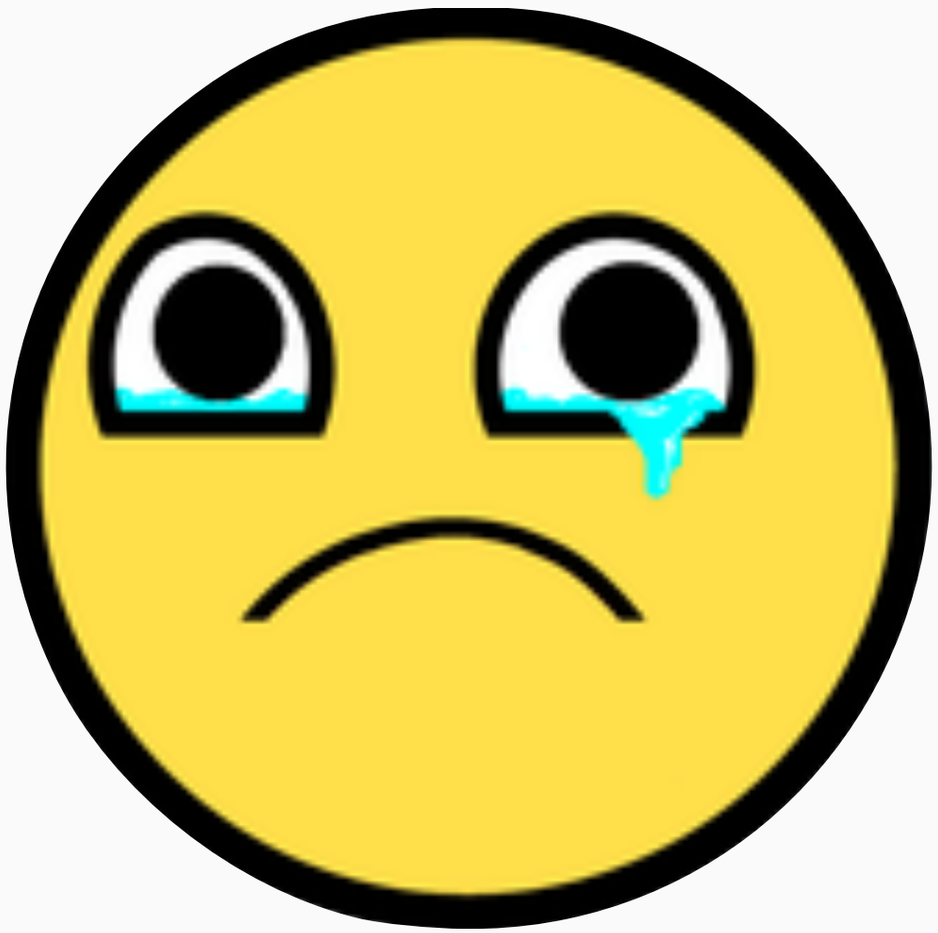
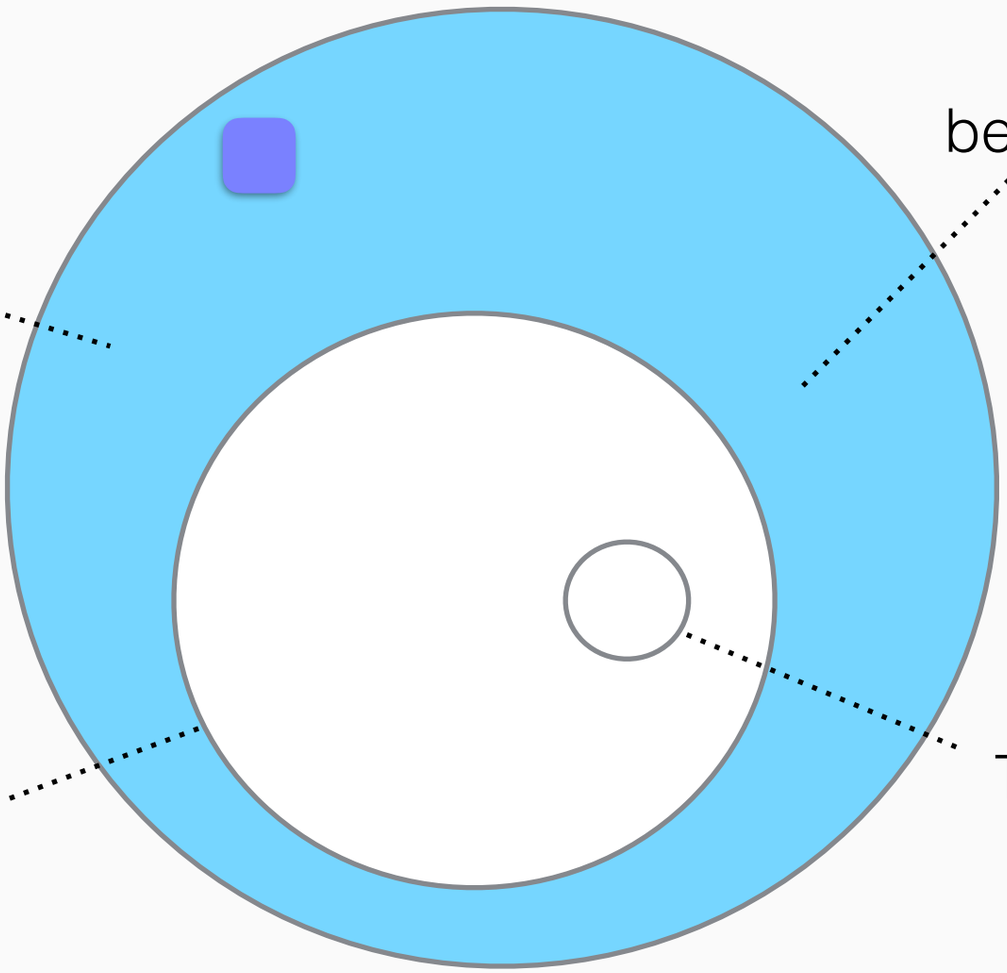


Q: We Can’t!

Computability

Q: What, if anything, is out here?



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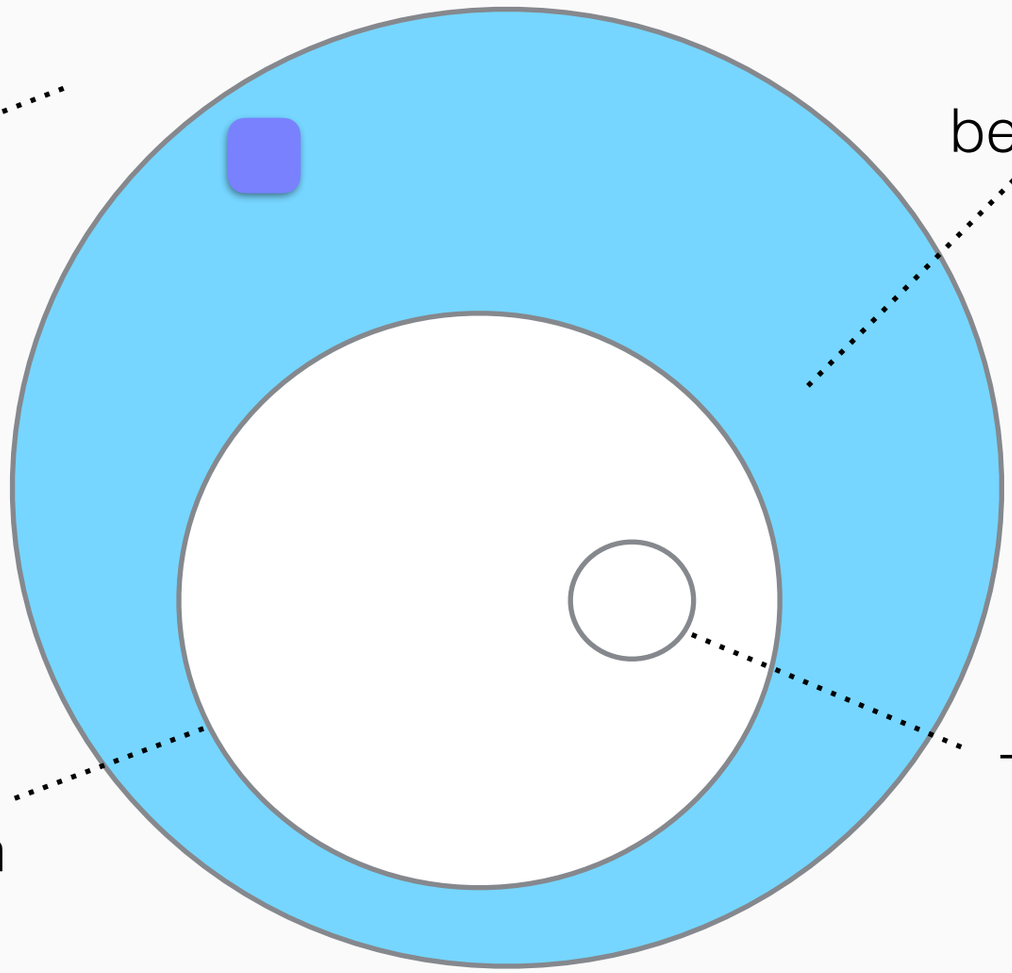

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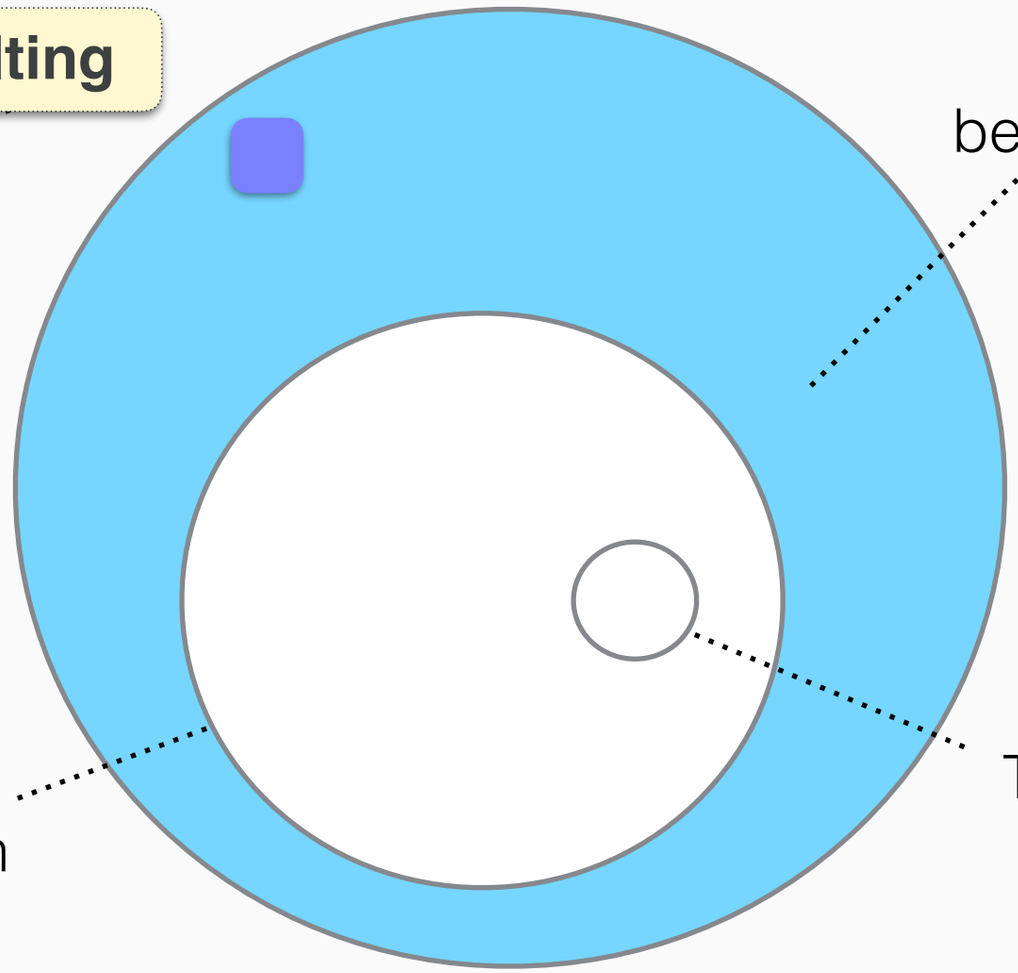

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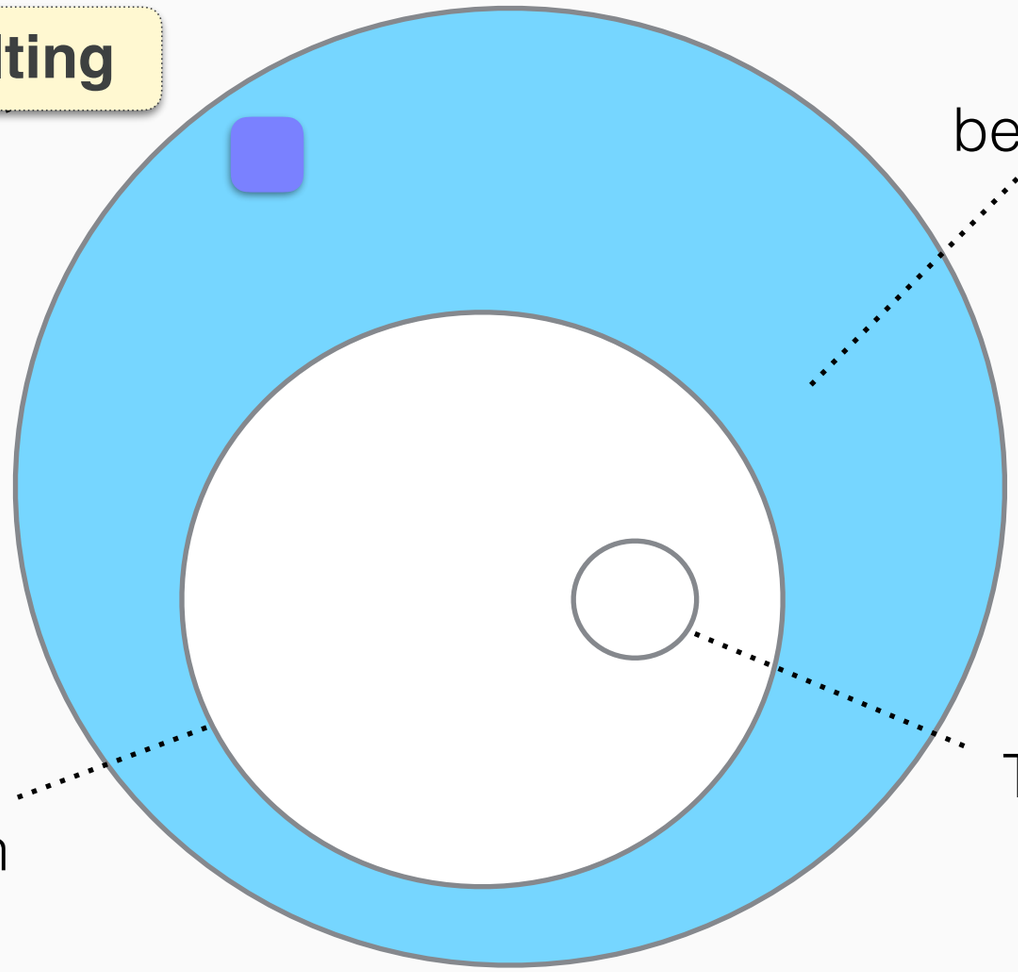

For more, see Unit 7!

Unit 3 Reflection

- An algorithm is a recipe for solving a problem.
- An algorithm has three properties of interest:
  - Does it halt?
  - Does it correctly solve the problem?
  - What is its growth rate? (How efficient is it?)
- Four Problems:
  - Search (algorithms = Random Search, Linear Search, Binary Search)
  - Sort (algorithms = Random Sort, Selection Sort)
  - Satisfiability (algorithms = Random Checker, Build The Truth Table)
  - Will it Halt? (algorithms = Loop Checker)