Unit “Naught”: Codes

Dave Abel

March 25th, 2016

Outline

- Why do we need codes?
- Solutions
  - Correcting Errors
  - Compression

Why Do We Need Codes?

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ERRORS

Cello

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Reason One: Come up with a *code* that is robust to errors!

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Why *else* Might We Need Codes?

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Why Do We Need Codes?

error

Reason One: Communication that is robust to errors!

Reason Two: Send information in *compressed form!*

compression

Error Correcting Codes

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- Idea: or what if we could at least *tell* our message broke?

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02906

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02906

First Idea:

- Send it twice! (or more)
- If they’re the same, great!
- If they’re different, ask to be sent the message again

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02905
02906

Clicker Question!

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Q: Is this scheme perfect? Will you *always* catch errors?

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First Idea:

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- If they're the same, great!
- If they're different, ask to be sent the message again
- Note: not perfect!
- The more copies we send, the better we do.
- But then we have to send way more stuff!

Second Idea: Checksum

- Idea: add some information to the message that can help detect errors!

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- Idea: add some information to the message that can help detect errors!
- Checksum: add the sum of the message *mod 10* to the end!
- Example: 02906, the sum is 0 + 2 + 9 + 0 + 6 = 17
- So we add 17 *mod 10* to the end, which is **029067**.
- Then check to make sure the checksum is still correct!

Clicker Question!

What is the checksum for 42304?

[A] 423041

[B] 423049

[C] 423047

[D] 423043

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Since: 4 + 2 + 3 + 4 = 13, and 13 mod 10 = 3

Clicker Question!

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5040

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5040 ⟶ 50409

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5040 ⟶ 50409 ⟶  ⟶ 50301

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Error Correcting Codes

- In a computer, communication errors happen regularly!
  - Errors in transmitting to and from the internet
  - Errors reading/writing from our computers memory
    - Old news: Errors reading/writing to CDs, floppies.
- We want to know when errors happen so we can fix them.
- Solutions:
  - Send copies! (But too much space)
  - Send a checksum!

Compression

- Idea: We can turn *big* things into *small* things that effectively *preserve the main information*.

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- Idea: We can turn *big* things into *small* things that effectively *preserve the main information*.

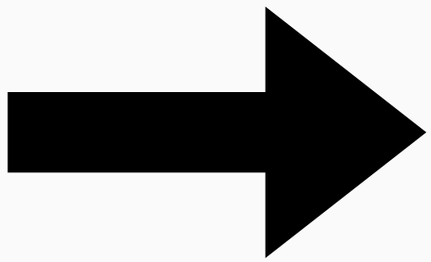
Compression

- Idea: We can turn *big* things into *small* things that effectively *preserve the main information*.

, but way bigger

Simple Explanations

Challenge 1:



Simple Explanations

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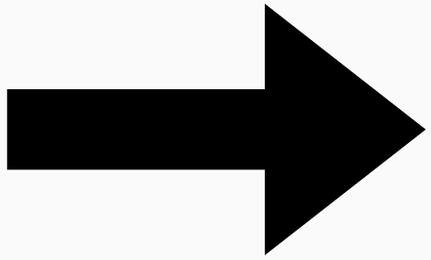
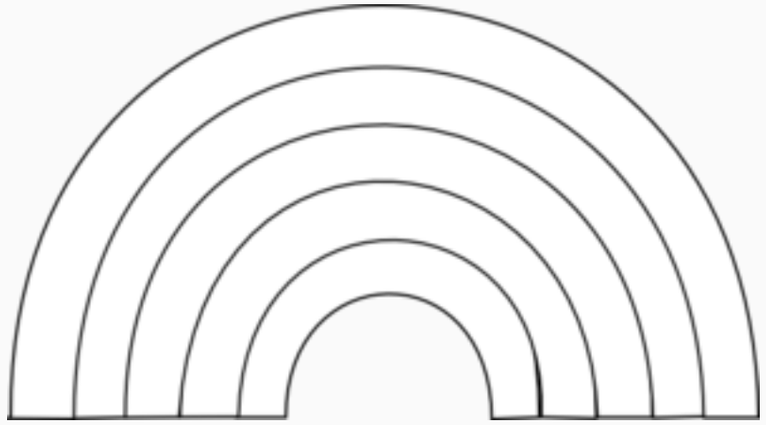


Shortest description: east

Simple Explanations

Challenge 2:



Simple Explanations

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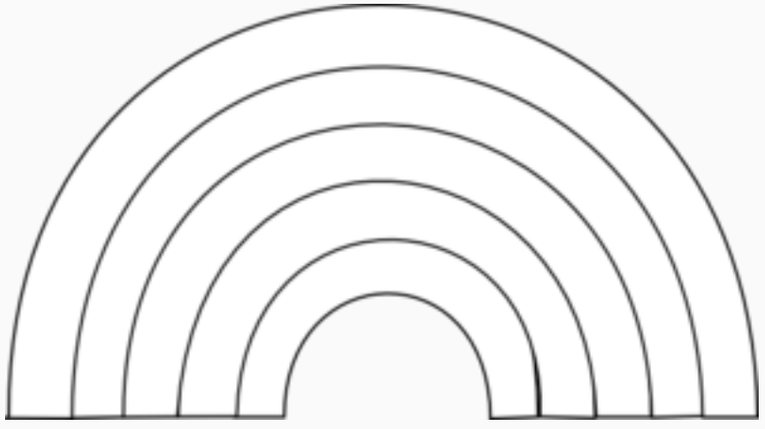
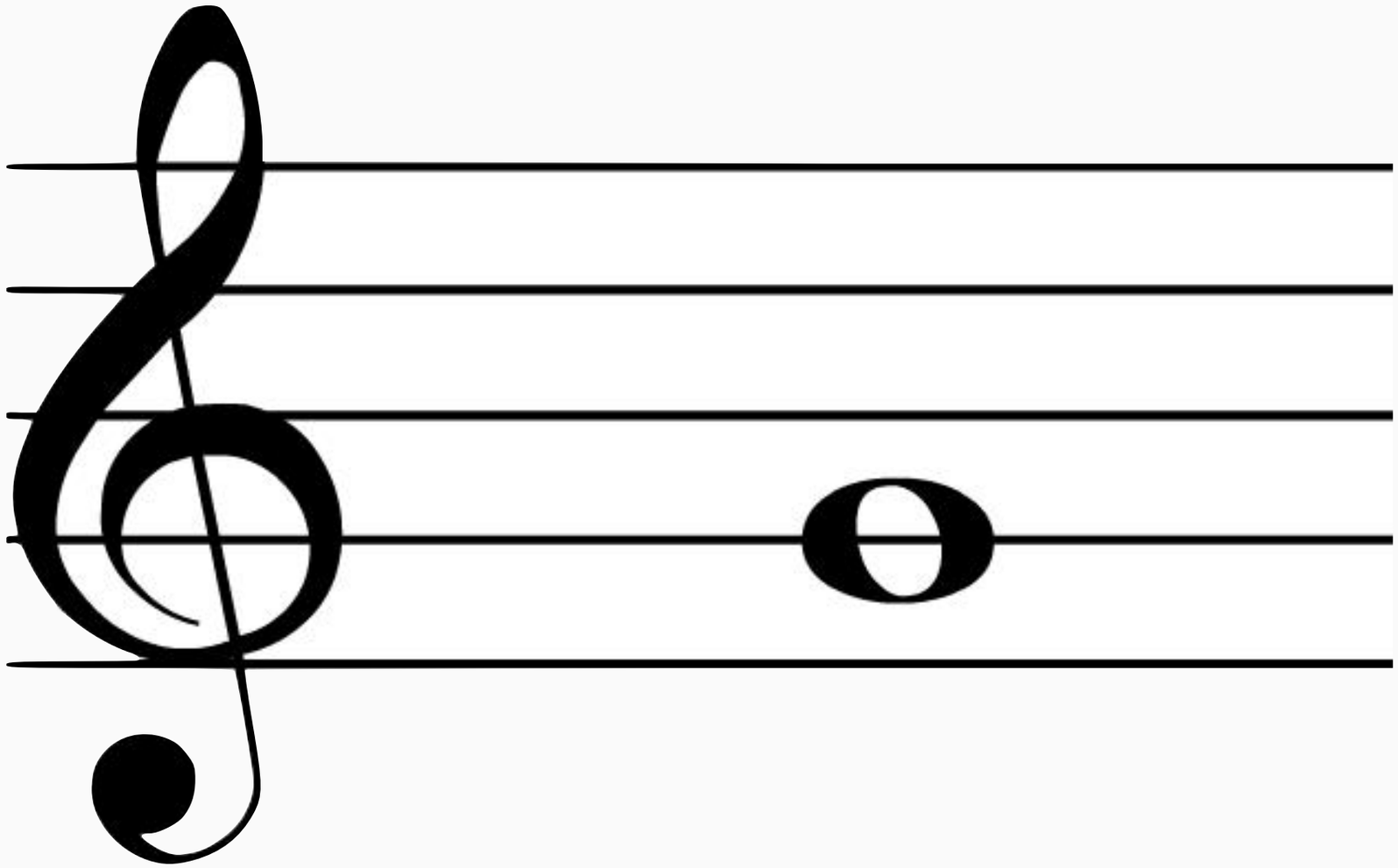


Shortest description: colorless rainbow

Simple Explanations

Challenge 4:



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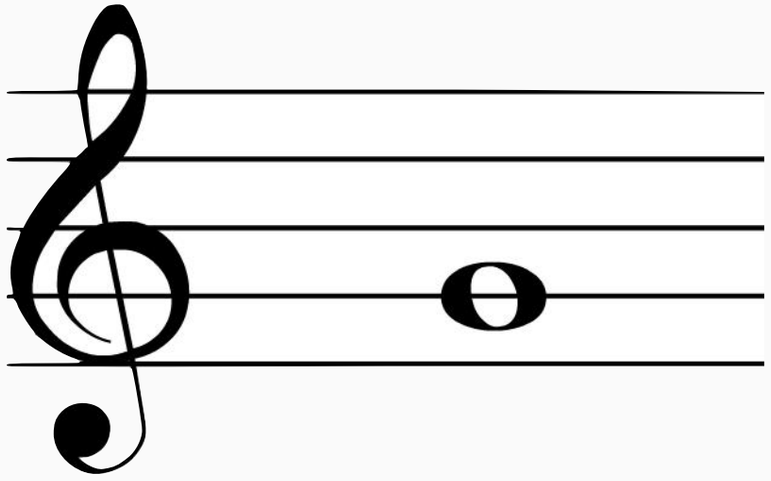
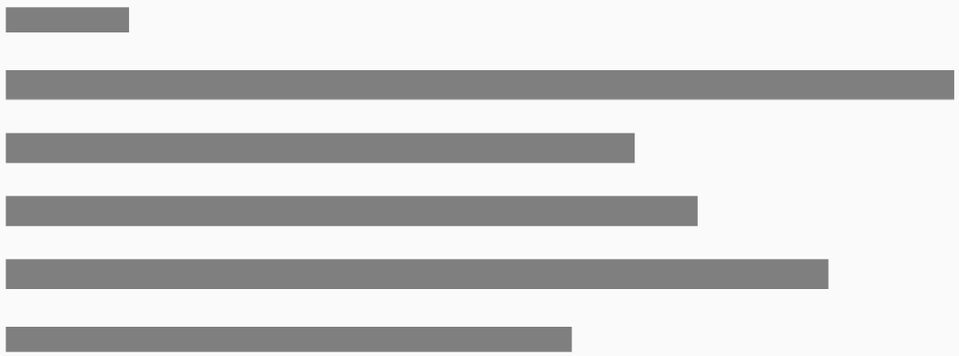


Shortest description: whole note g

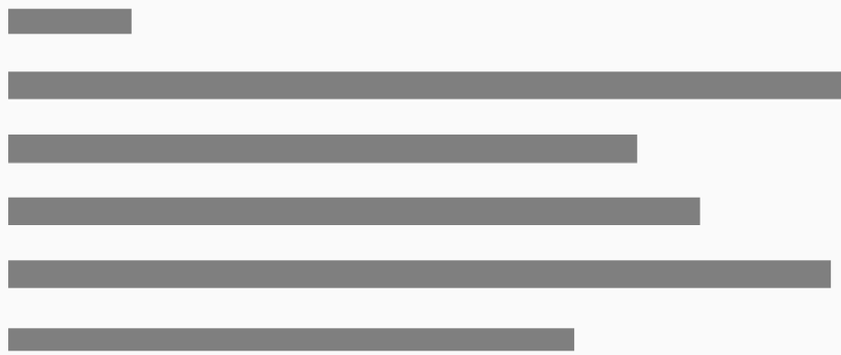
Simple Explanations

Challenge 5:



Simple Explanations

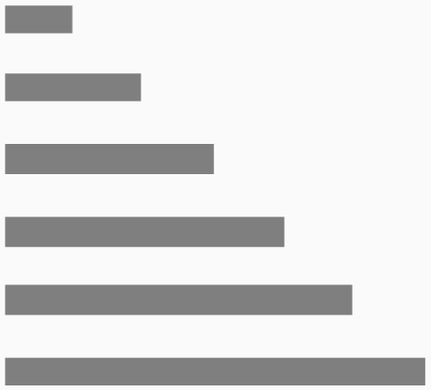
Challenge 5:



Shortest description: six bar histogram switch 2 and 6

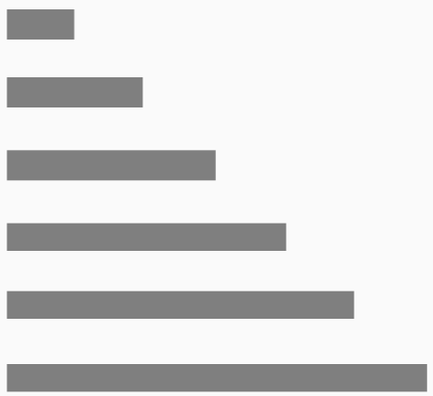
Simple Explanations

Challenge 6:



Simple Explanations

Challenge 6:



Shortest description: bars ordered

Compression

- Q: How can we make an object *as small as possible*, but still preserve what the object is?

, but way bigger

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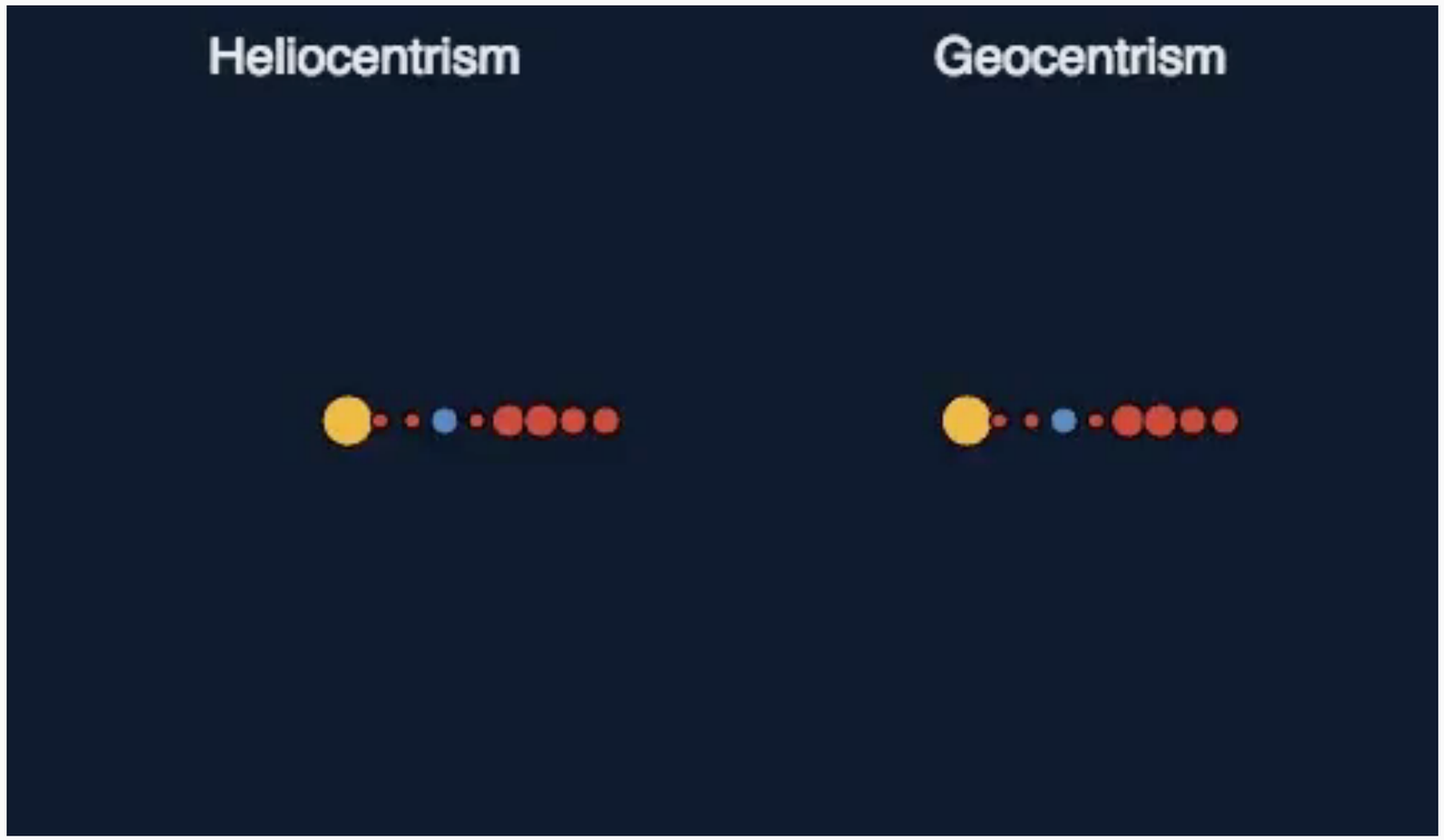
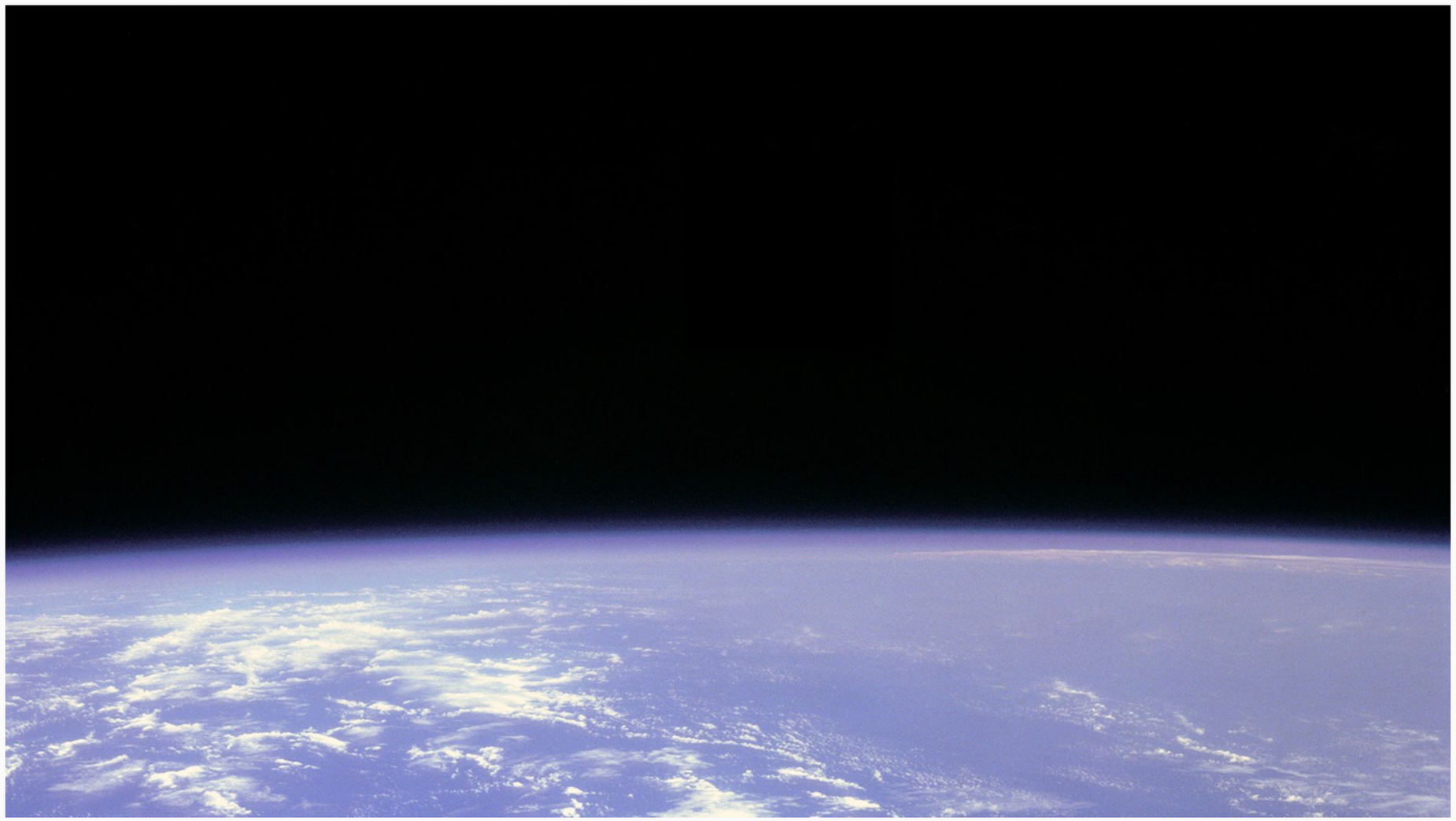
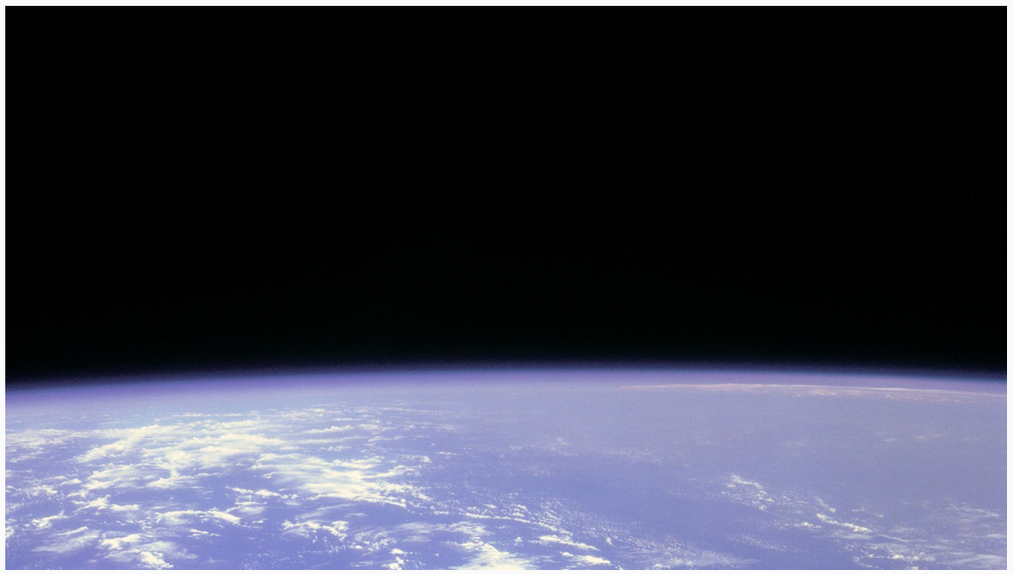


Compression: Intuition



Compression: Intuition



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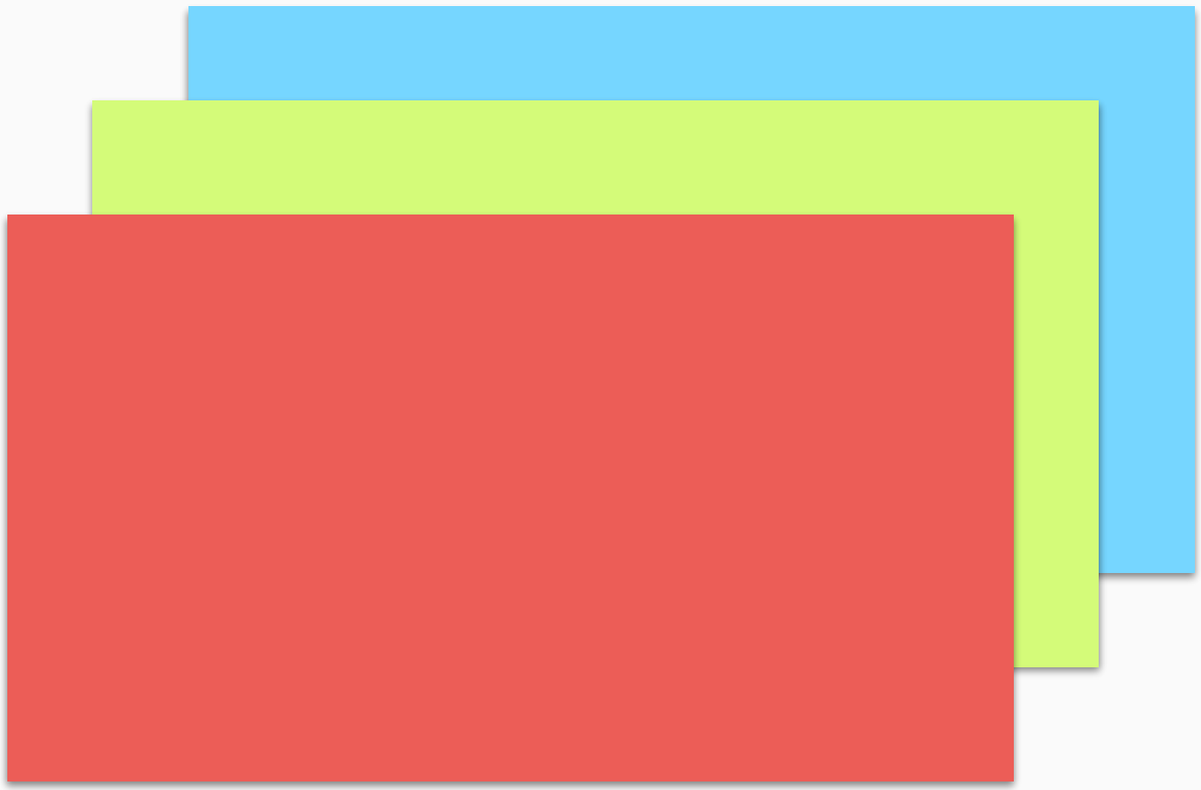
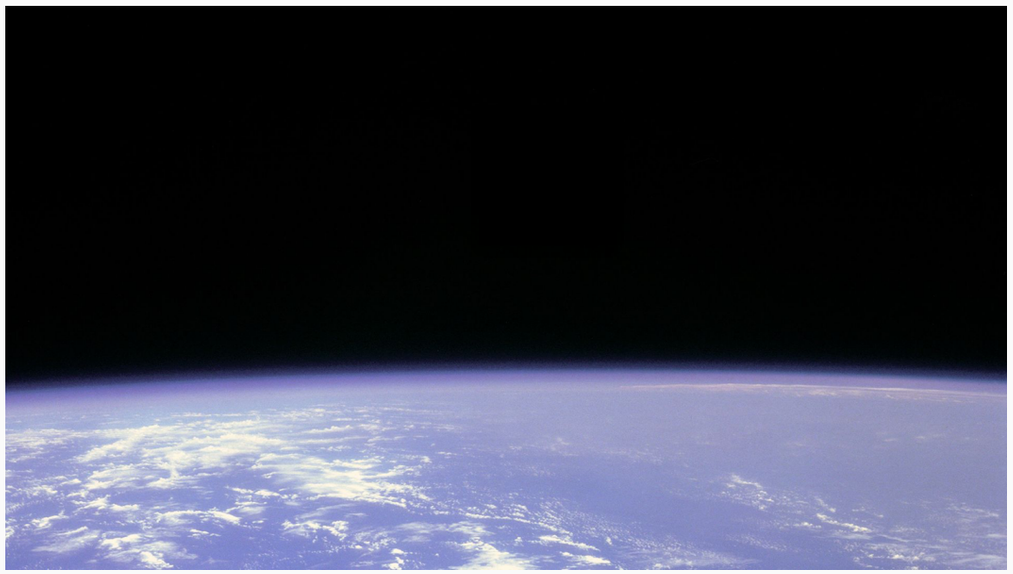


Compression: Intuition



1Kb

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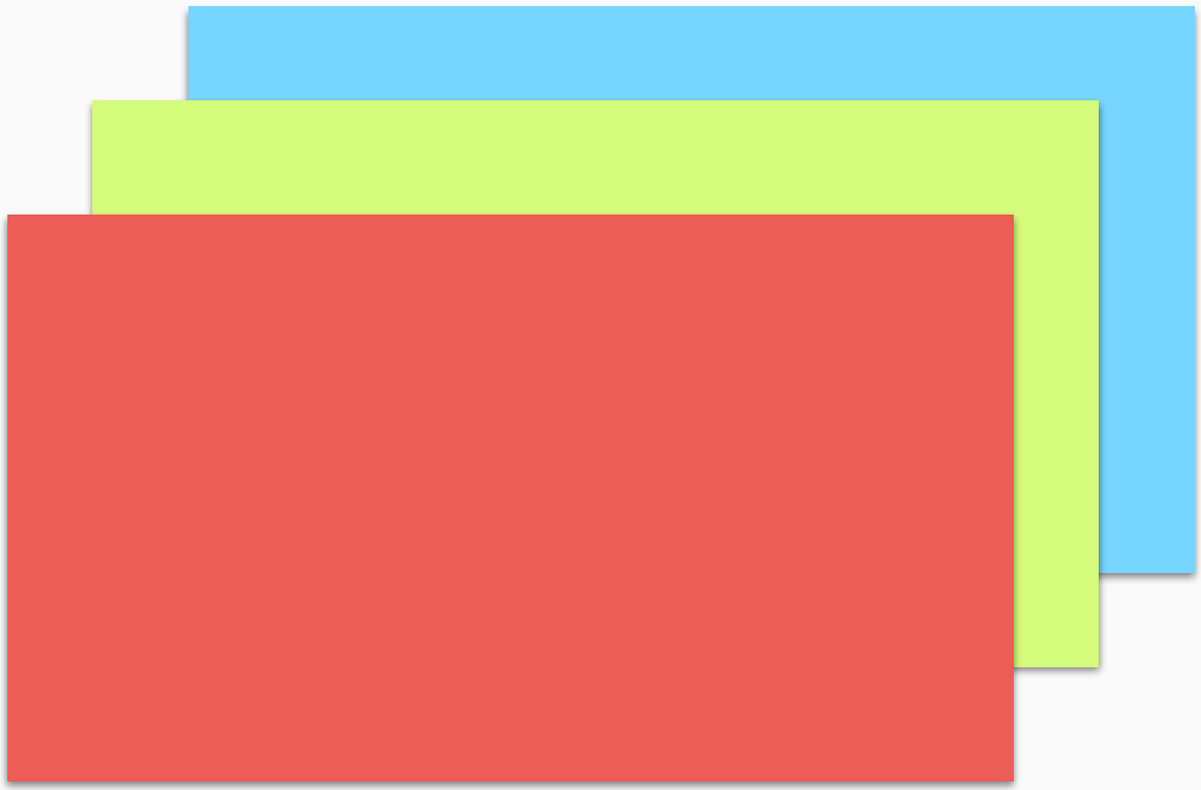

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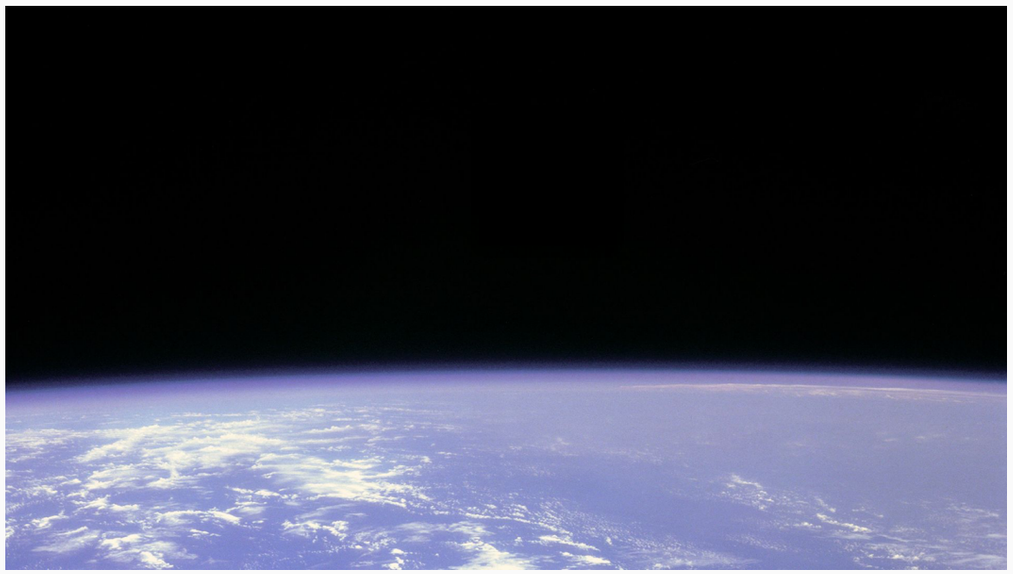

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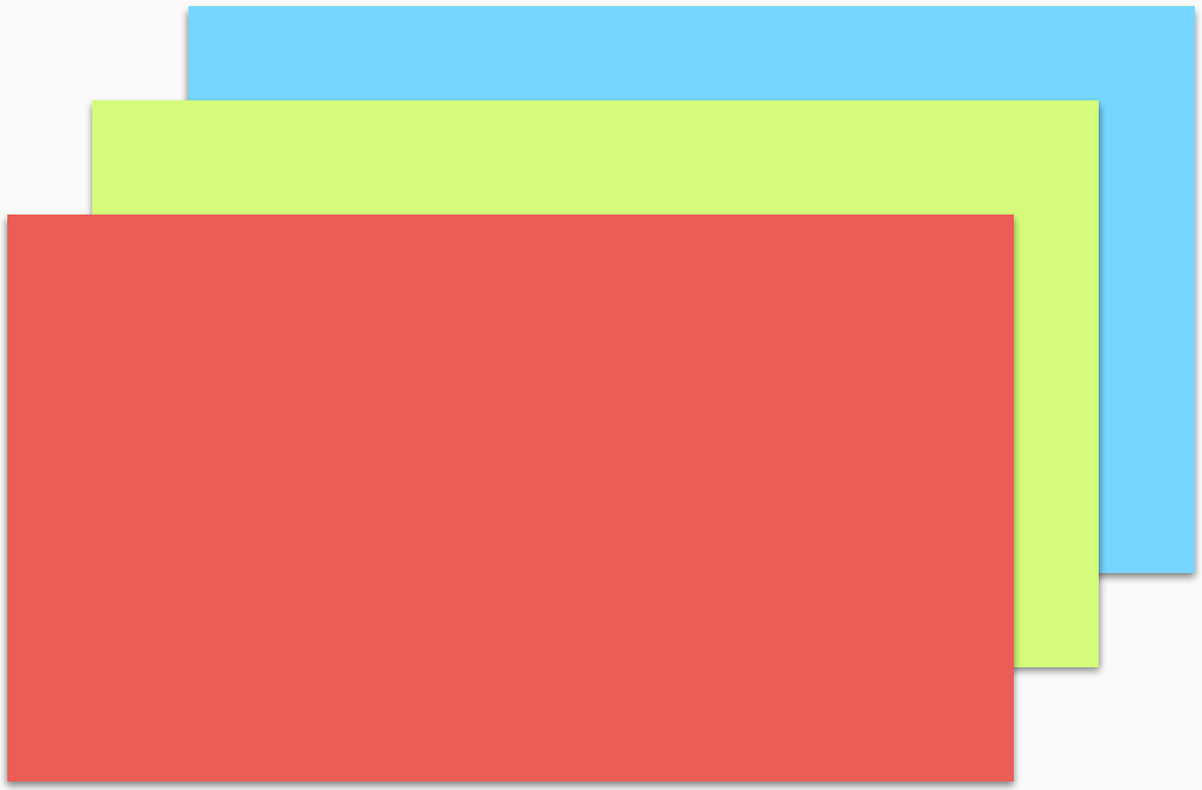
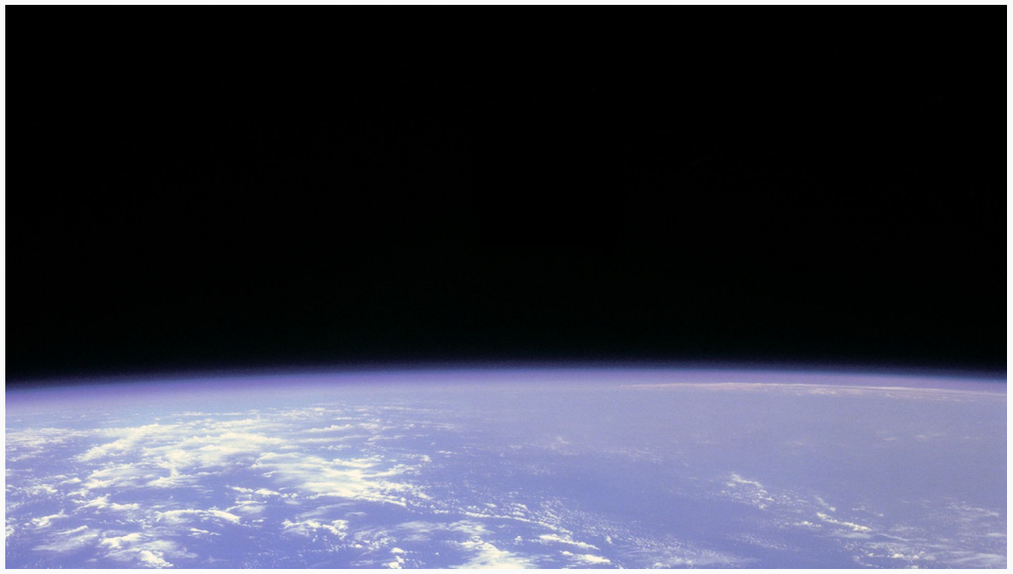


1Kb

Compression: Intuition

Everything else, black



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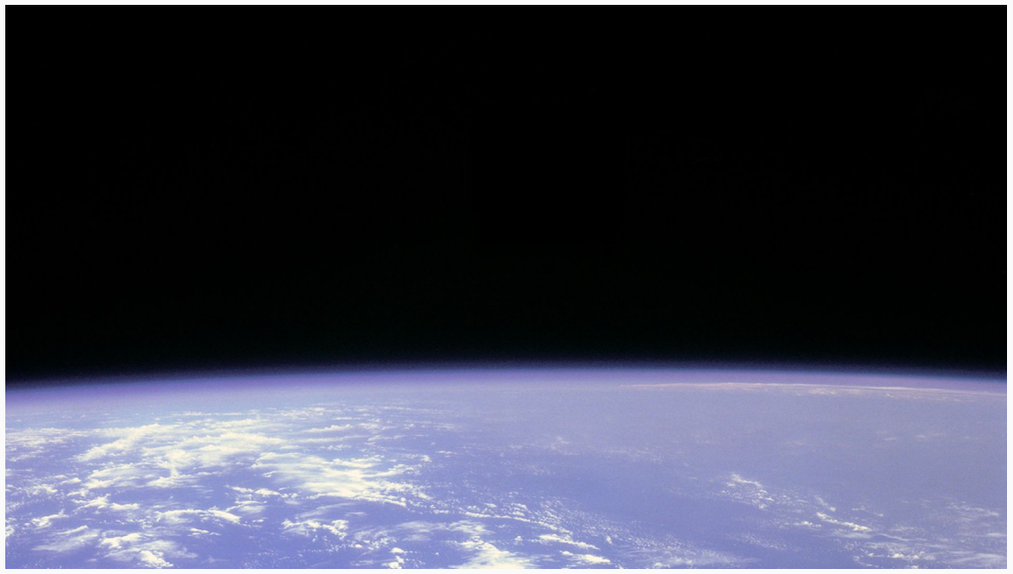
0.3Kb

Compression: Intuition



1Kb

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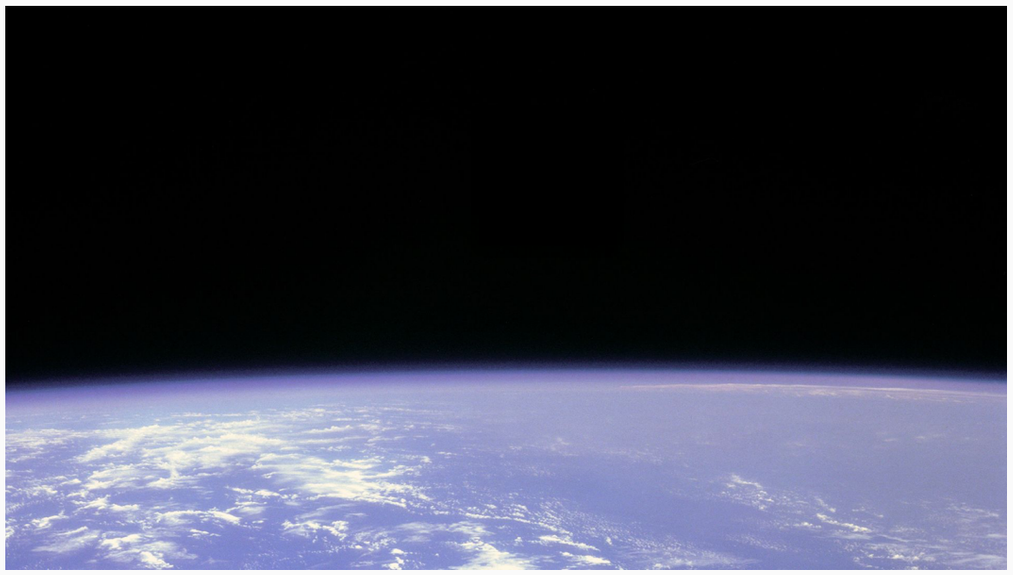
Everything else, black

0.3Kb

Shaved off .7 Kilobytes!

Compression



1Kb

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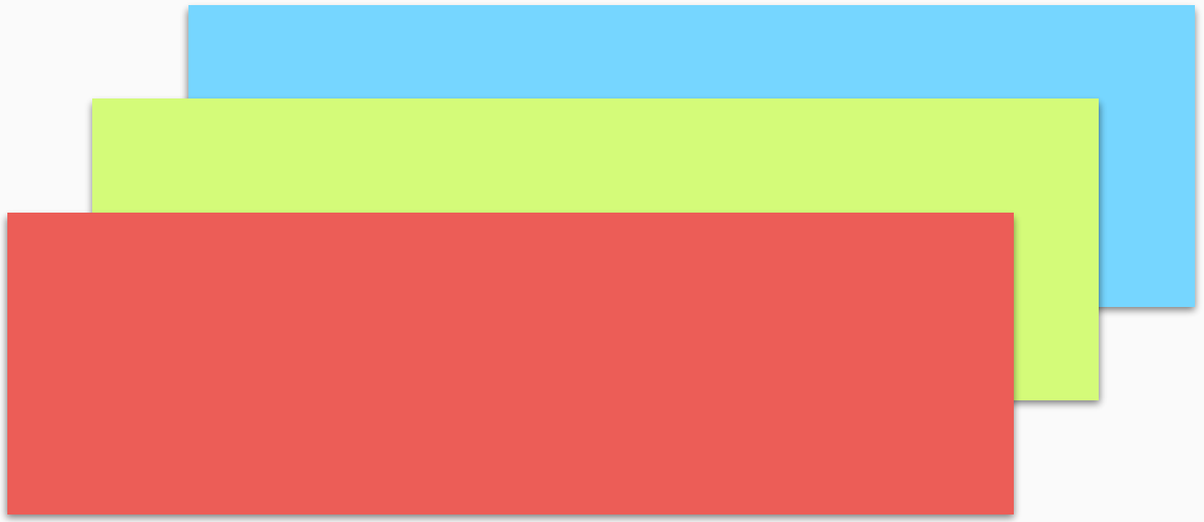
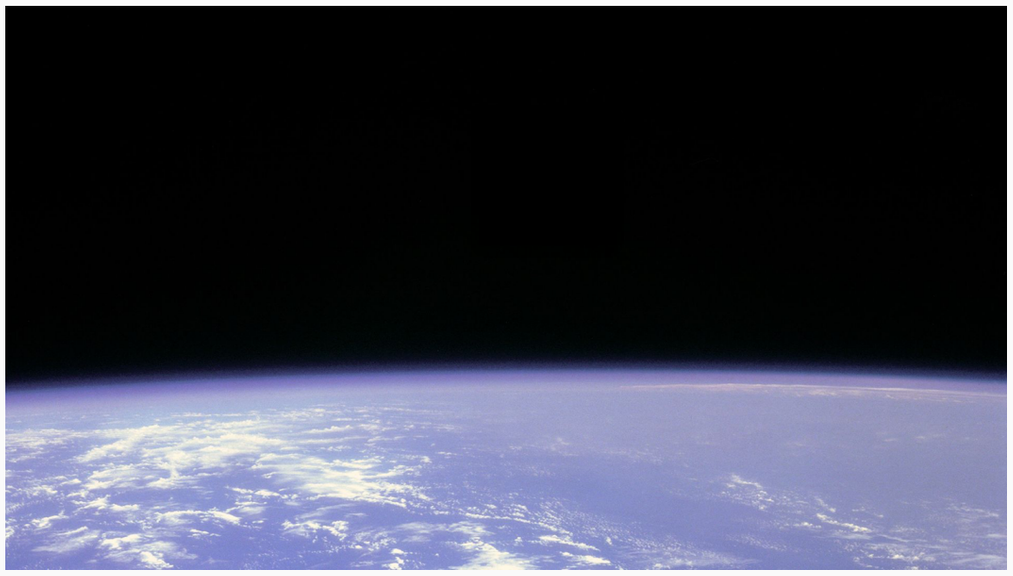


0.3Kb

Critical: we can do this with everything

Clicker Question:

Everything else, black



1Kb

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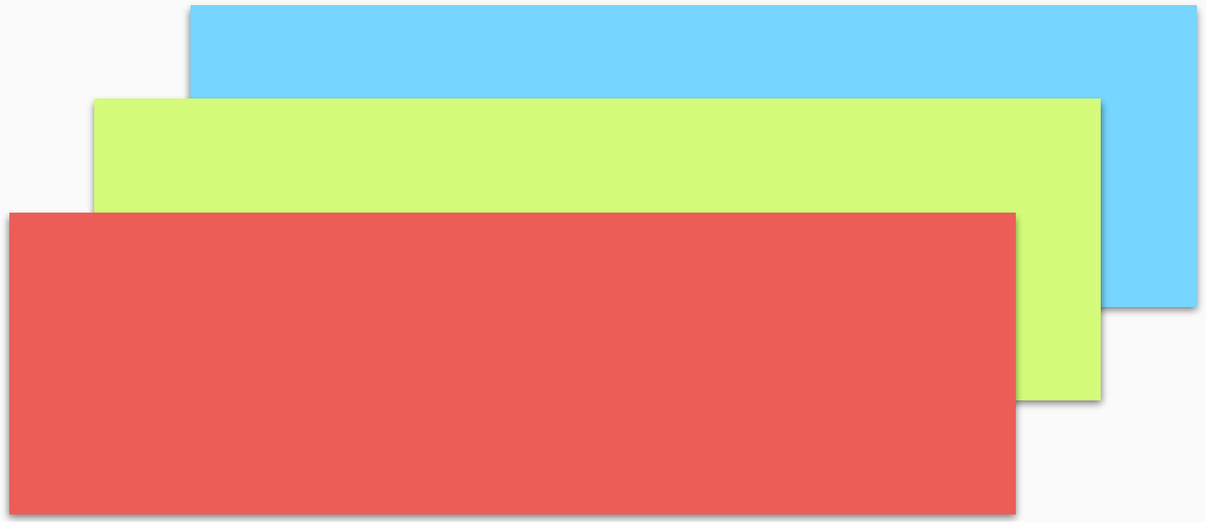

0.3Kb

Critical: we can do this with everything

Algorithmic Information

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For a given object…

Q: What is the shortest algorithm that outputs the object we want to describe?

Algorithmic Information

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Let’s consider sequences of english characters…

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Example: Scooooooooooooooooooooooby doo!

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Shaggy's algorithm:
Output "Sc", then 20 "o"'s, then "by doo!"

Algorithmic Information

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Let's consider sequences of english characters…

Example: aaaaaaaaaaaaaaaaaaaaaaaaa

Shaggy's algorithm:
Output 23 "a"

Problem: Compress Words

- INPUT: A phrase in english
- OUTPUT: A compressed version of that phrase.

Algorithm: Run Length Encoding

- Compress repeated sequences into #repeats, then the letter:
  - E.g. “fooooooooooxesssss!”
    - Becomes: 1f11o1x1e5s
  - E.g. “abbbbbaa zabbbaa”
    - Becomes: 1a5b2a 1z1a3b2a

Clicker Question!

Q: What is the run length encoding for “Wobbbless and bbbbbles”?

[A] 1Wobbless 1and 1bbbbbles

[B] 1W1o3b1l1e2s 1a1n1d 5b1l1e1s

[C] Wo3b1l1e2s and 5bles

[D] I’m confused

Clicker Answer!

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Clicker Question!

Q: What is, “1W5o1d1e1n 1B3i1r1d1s”, when *uncompressed*?

[A] Wooooooden Biiirds

[B] Wooden Birds

[C] Wooden Biiirds

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What Do We Think of This?

- Compress repeated sequences into #repeats, then the letter:
  - E.g. “fooooooooooxesssss!”
    - Becomes: 1f11o1x1e5s
  - E.g. “abbbbbaa zabbbaa”
    - Becomes: 1a5b2a 1z1a3b2a

What Do We Think of This?

- Two drawbacks of note:
  - Q: Are repeated sequences that regular in English?
  - Q: What about numbers?

Algorithmic Information

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This question ends up being *super fascinating*.

Algorithmic Information

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This question ends up being *super fascinating*.

Q: What objects generally have shorter algorithms?

Q: What objects generally have complex algorithms?

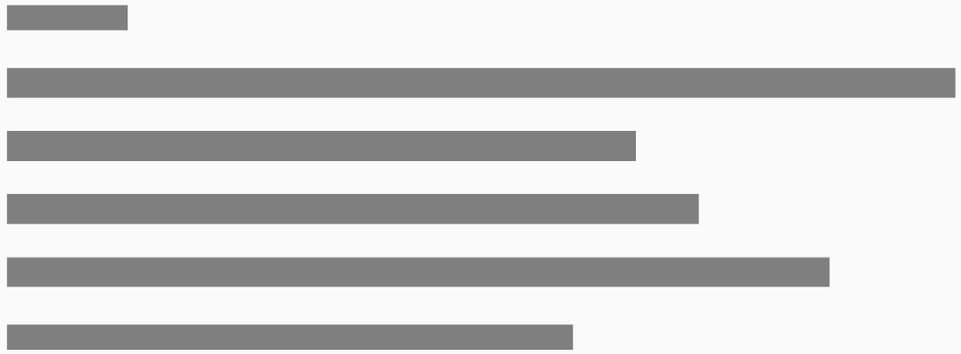
Q: Given an object, how can *find* the shortest algorithm?

Algorithmic Information

Q: What is the shortest algorithm that outputs the object we want to describe?

This question ends up being *super fascinating*.

Q: What objects generally have more complex algorithms?



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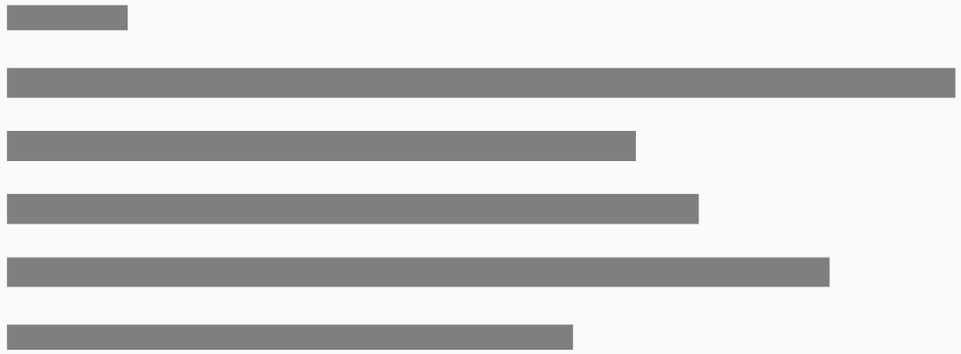

Have a Great Break!