Unit 1:
AND(Logic,Gates)

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February 3rd, 2016

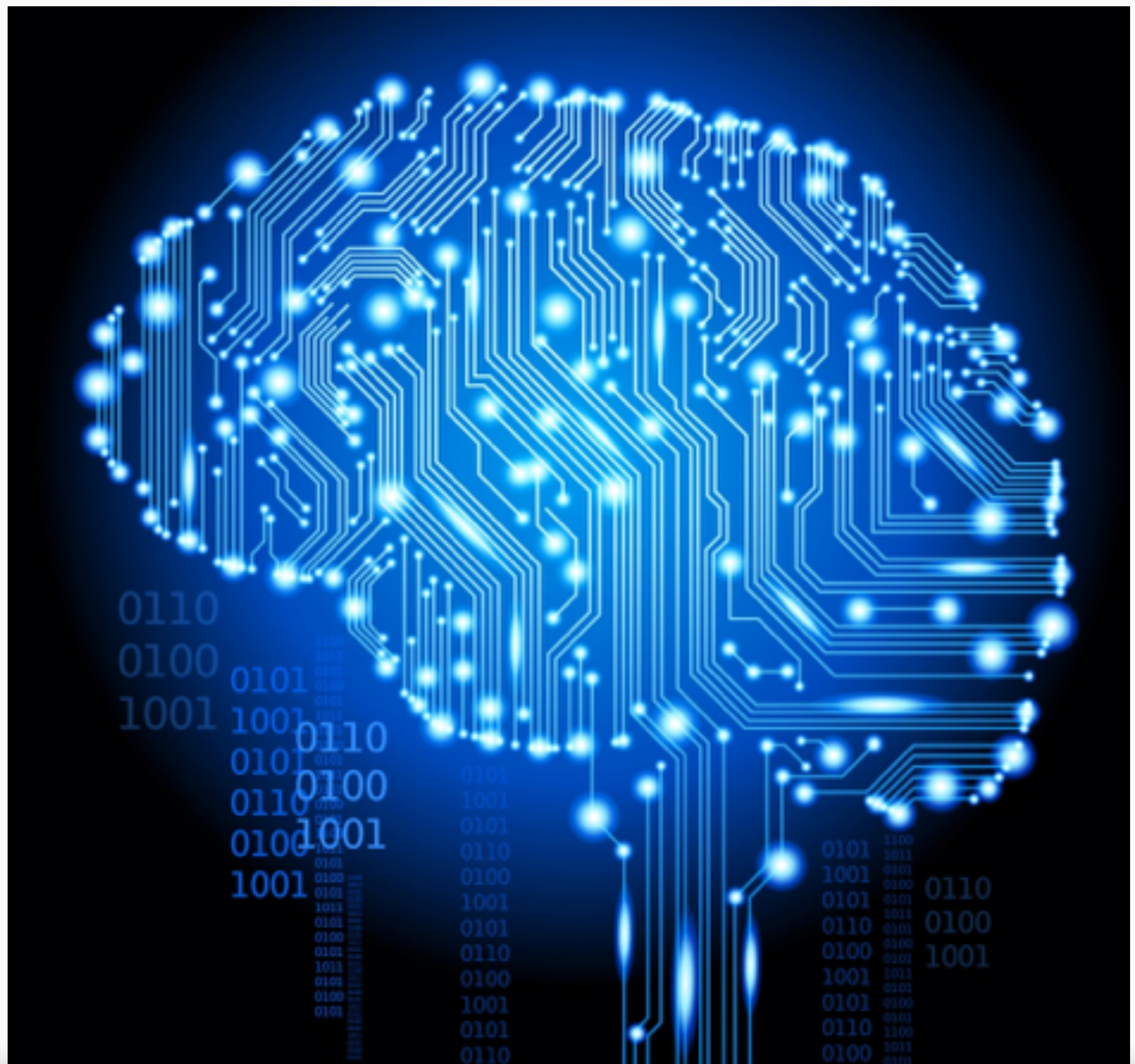
Today's Takeaway

- Monday's claim: Computers are doing logic!
- Today: *how* they do logic! (physically!)

Outline

- Logic review
  - Boolean Sentences
  - Logical Functions
  - Truth Tables
- Gates

Still Need: Reasoning



Logic: A *Formal* Language

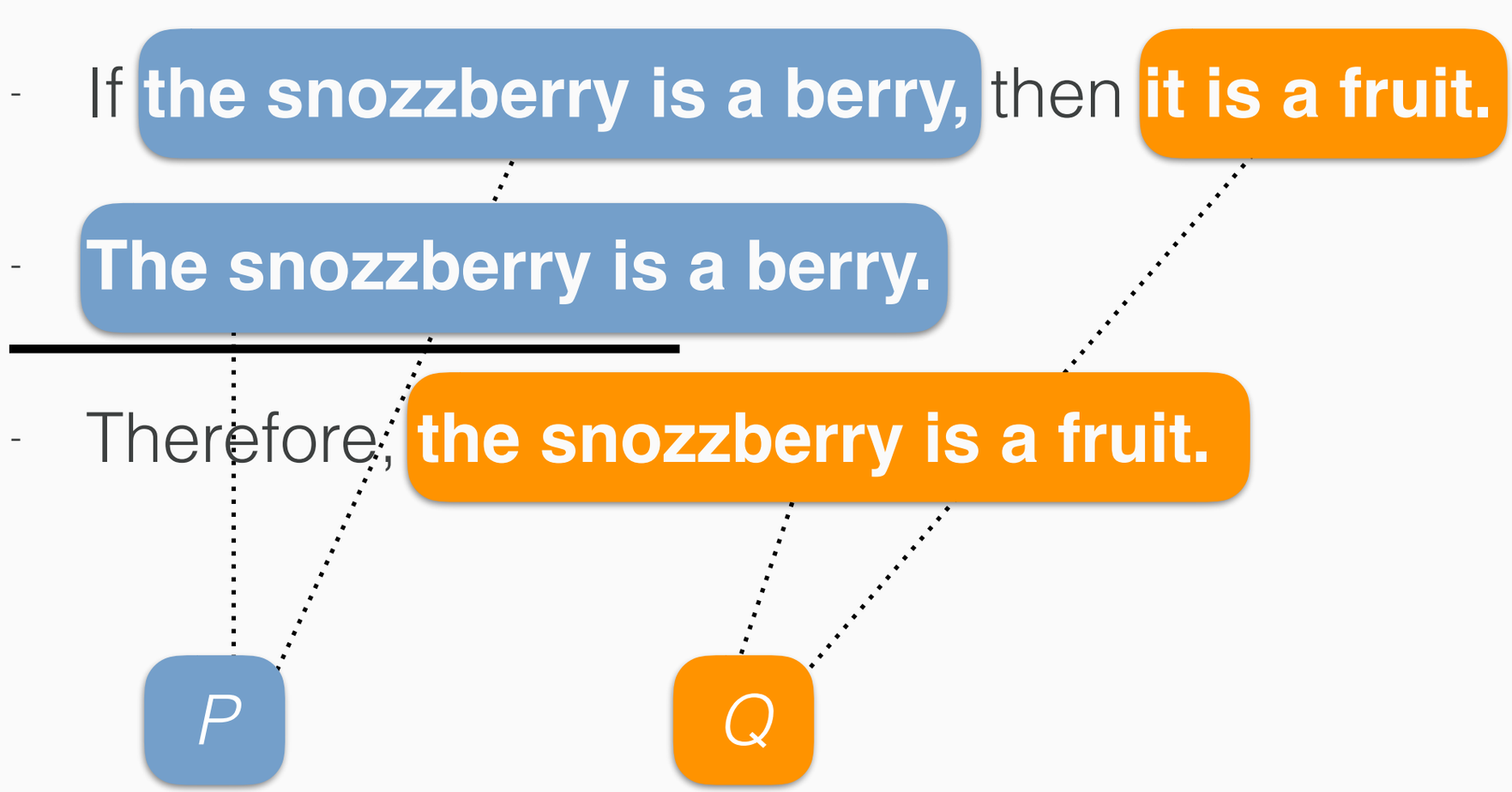
- **Variables that stand for sentences:** *P, Q, R, S*
- Example:

- If the snozzberry is a berry, then it is a fruit.
- The snozzberry is a berry.

- Therefore, the snozzberry is a fruit.

Logic: A *Formal* Language

- **Variables that stand for sentences:** *P, Q, R, S*
- Example:



Logic: A *Formal* Language

- **Variables that stand for sentences:** *P, Q, R, S*
- Example:
  - If *P* then *Q*
  - *P*

  - Therefore, *Q*

P = The snozzberry is a berry.

Q = The snozzberry is a fruit.

Logic: A *Formal* Language

- **Variables that stand for sentences:** *P, Q, R, S*
- Example:
  - If *P* then *Q*
  - *P*

  - Therefore, *Q*

True for all sentences *P*,

All sentences *Q!*

Logic: A *Formal* Language

- **Variables that stand for sentences:** *P, Q, R, S*
- Example:
  - If *P* then *Q*
  - *P*
  - Therefore, *Q*

Premises: assume to be true.

Logic: A *Formal* Language

- **Variables that stand for sentences:** *P, Q, R, S*
- We call sentences that can be True or False "**Boolean**".
- So: *P, Q, R, S,* etc., will be called **Boolean Sentences.**

Logic: Boolean Functions

- We get three functions: *AND, OR, NOT*

  - Each function takes as input one or more Boolean Sentences (*P, Q,* etc.)

  - Outputs a Boolean value (True, False)

Logic: Boolean Functions

AND(P,Q)

Outputs True if **both** *P* and *Q* are True.

OR(P,Q)

Outputs True if **at least one of** *P* or *Q* is True.

NOT(P)

Outputs True if *P* is False. (Just flips it!)

Truth Tables: *NOT*

P	*NOT(*	*P)*
T	**F**	T
F	**T**	F

Truth Tables: *AND*

P	*Q*	*AND*(	*P*	*Q*)
T	T	**T**	T	T
T	F	**F**	T	F
F	T	**F**	F	T
F	F	**F**	F	F

Truth Tables: *OR*

P	*Q*	*OR*(	*P*	*Q*)
T	T	**T**	T	T
T	F	**T**	T	F
F	T	**T**	F	T
F	F	**F**	F	F

Logic: Composition

- Boolean Sentences represented with a letter are called **Atomic Sentences** (e.g. *P, Q, R, S,* etc.)

- But since *AND(-,-), OR(-,-),* and *NOT(-)*, also output Boolean Values, they are *also Boolean Sentences.*

- For example:

  - *AND(NOT(P),Q)*

  - *OR(AND(P,Q),NOT(R))*

Truth Tables: *Composite*

P	*Q*	*OR(*	*NOT(*	*Q),*	P)
T	T				
T	F				
F	T				
F	F				

Truth Tables: *Composite*

P	*Q*	*OR(*	*NOT(*	*Q),*	P)
T	T			T	T
T	F			F	T
F	T			T	F
F	F			F	F

Truth Tables: *Composite*

P	*Q*	*OR(*	*NOT(*	*Q*),	P)
T	T		F	T	T
T	F		T	F	T
F	T		F	T	F
F	F		T	F	F

Truth Tables: *Composite*

P	*Q*	*OR(*	*NOT(*	*Q),*	P)
T	T	**T**	F	T	T
T	F	**T**	T	F	T
F	T	**F**	F	T	F
F	F	**T**	T	F	F

Logical Rules!

Logical Rules!

When is it okay to attend a rated R movie?

Logical Rules!

When is it okay to attend a rated R movie?

P: person X is 17 or older

Q: person X is accompanied by a parent/adult guardian

Logical Rules!

When is it okay to attend a rated R movie?

P: person X is 17 or older

Q: person X is accompanied by a parent/adult guardian

OR(P,Q)

Logical Rules! You Try It

Exercise: write a rule for a person being eligible to receive free shipping from “Alabamazon”, which provides free shipping according to the following rules:

1. You receive free shipping on orders above $60.

2. You receive free shipping if you do not order any premium items.

Logical Rules! You Try It

Exercise: write a rule for a person being eligible to receive free shipping from “Alabamazon”, which provides free shipping according to the following rules:

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P = order is above $60

Q = ordered a premium item

Logical Rules! You Try It

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P = order is above $60

Q = ordered a premium item

OR(P,NOT(Q))

Logical Rules! You Try It

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3. You receive free shipping if you’re shipping to Alabama and your order is less than $60.

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P = order is above $60

Q = ordered a premium item

R = shipped to alabama

Logical Rules! You Try It

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R = shipped to alabama

Previous example: *OR(P,NOT(Q))*

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P = order is above $60

Q = ordered a premium item

R = shipped to alabama

AND(R,NOT(P))

Logical Rules! You Try It

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2. You receive free shipping if you do not order any premium items.

3. You receive free shipping if you’re shipping to Alabama and your order is less than $60.

P = order is above $60

Q = ordered a premium item

R = shipped to alabama

AND(R,NOT(P))

OR(P,NOT(Q))

Logical Rules! You Try It

Exercise: write a rule for a person being eligible to receive free shipping from “Alabamazon”, which provides free shipping according to the following rules:

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2. You receive free shipping if you do not order any premium items.

3. You receive free shipping if you’re shipping to Alabama and your order is less than $60.

P = order is above $60

Q = ordered a premium item

R = shipped to alabama

AND(R,NOT(P)) or *OR(P,NOT(Q))*

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1. You receive free shipping on orders above $60.

2. You receive free shipping if you do not order any premium items.

3. You receive free shipping if you’re shipping to Alabama and your order is less than $60.

P = order is above $60

Q = ordered a premium item

R = shipped to alabama

OR(OR(P,NOT(Q)),AND(R,NOT(P))

Logical Rules!

OR(P,Q)

P or *Q*

AND(P,Q)

P and *Q*

NOT(P)

not *P*

Logical Rules!

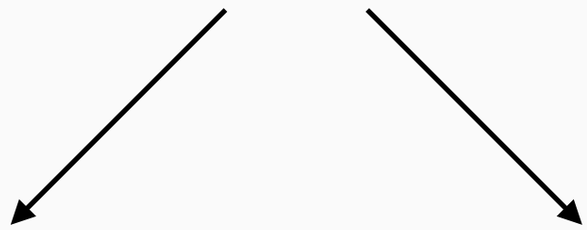
Why shouldn't we do this:

P or *Q* and *R*

Logical Rules!

Why shouldn't we do this:

P or *Q* and *R*



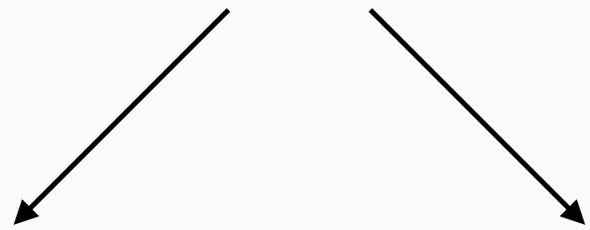
(P or *Q)* and *R*

P or (*Q* and *R)*

Logical Rules!

Why shouldn't we do this:

~~*P* or *Q* and *R*~~



(P or *Q)* and *R*

AND(R,OR(P,Q))

P or (*Q* and *R)*

OR(P,AND(R,Q))

Truth Table to Formula

P	*???*
T	**T**
F	**T**

Truth Table to Formula

P	*???*
T	**T**
F	**T**

Q: What rule goes in the ???

Truth Table to Formula

P	*???*
T	**T**
F	**T**

Q: What rule goes in the ???

Strategy:

1. Make a rule for each “True”
2. OR them together

Truth Table to Formula

P	*???*
T	**T**
F	**T**

Q: What rule goes in the ???

Strategy:

1. Make a rule for each “True”
2. OR them together

P

NOT(P)

Truth Table to Formula

P	*???*
T	**T**
F	**T**

Q: What rule goes in the ???

Strategy:

1. Make a rule for each “True”
2. OR them together

NOT(P) or *P*

Truth Table to Formula

P	*???*
T	**T**
F	**T**

Q: What rule goes in the ???

Strategy:

1. Make a rule for each “True”
2. OR them together

OR(*NOT(P)* , *P*)

Truth Table to Formula

P	*Q*	*???*
T	T	**F**
T	F	**T**
F	T	**F**
F	F	**T**

Q: What rule goes in the ???

Strategy:

1. Make a rule for each “True”
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Truth Table to Formula

P	*Q*	*???*
T	T	**F**
T	F	**T**
F	T	**F**
F	F	**T**

Q: What rule goes in the ???

AND(P,NOT(Q))

AND(NOT(P),NOT(Q))

Strategy:

1. Make a rule for each “True”
2. OR them together

Truth Table to Formula

P	*Q*	*???*
T	T	**F**
T	F	**T**
F	T	**F**
F	F	**T**

Q: What rule goes in the ???

AND(P,NOT(Q))

or

AND(NOT(P),NOT(Q))

Strategy:

1. Make a rule for each “True”
2. OR them together

Truth Table to Formula

P	*Q*	*???*
T	T	**F**
T	F	**T**
F	T	**F**
F	F	**T**

Q: What rule goes in the ???

OR(AND(P,NOT(Q)) ,
AND(NOT(P),NOT(Q)))

Strategy:

1. Make a rule for each “True”
2. OR them together

Truth Table to Formula

P	*Q*	*???*
T	T	**F**
T	F	**T**
F	T	**F**
F	F	**T**

Q: What rule goes in the ???

OR(AND(P,NOT(Q)) ,
AND(NOT(P),NOT(Q)))

Also: *NOT(Q)*

Strategy:

1. Make a rule for each “True”
2. OR them together

Truth Table to Formula

Q: Can we write down every possible logical formula in this way?

Truth Table to Formula

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A: YES!

Truth Table to Formula

Q: Can we write down every possible logical formula in this way?

A: YES!

Strategy:

1. Make a rule for each “True”
2. OR them together

Truth Table to Formula

Q: Can we write down every possible logical formula in this way?

A: YES!

Strategy:

1. Make a rule for each “True”
2. OR them together

Q: What if we only had *AND?*

Truth Table to Formula

Q: What if we only had *AND?*

A: No! Can't do this one:

P	*???*
T	**F**
F	**T**

All Logical Formulas!

Idea: with a certain set of logical functions, we can represent all possible logical formulas!

All Logical Formulas!

If *P*, then *Q*

All Logical Formulas!

If *P*, then *Q*

$$P \longrightarrow Q$$

Q: Can we represent this as a logical formula?

All Logical Formulas!

Q: Can we represent “If *P*, then *Q*” as a logical formula?

P	*Q*	*P*	→	*Q*
T	T	T		T
T	F	T		F
F	T	F		T
F	F	F		F

All Logical Formulas!

Q: Can we represent “If *P*, then *Q*” as a logical formula?

P	*Q*	*P*	→	*Q*
T	T	T		T
T	F	T		F
F	T	F		T
F	F	F		F

All Logical Formulas!

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P	*Q*	*P*	→	*Q*
T	T	T	T	T
T	F	T		F
F	T	F		T
F	F	F		F

All Logical Formulas!

Q: Can we represent “If *P*, then *Q*” as a logical formula?

P	*Q*	*P*	→	*Q*
T	T	T	T	T
T	F	T		F
F	T	F		T
F	F	F		F

All Logical Formulas!

Q: Can we represent “If *P*, then *Q*” as a logical formula?

P	*Q*	*P*	→	*Q*
T	T	T	T	T
T	F	T	F	F
F	T	F		T
F	F	F		F

All Logical Formulas!

Q: Can we represent “If *P*, then *Q*” as a logical formula?

P	*Q*	*P*	→	*Q*
T	T	T	T	T
T	F	T	F	F
F	T	F		T
F	F	F		F

All Logical Formulas!

Q: Can we represent “If *P*, then *Q*” as a logical formula?

P	*Q*	*P*	→	*Q*
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F		F

All Logical Formulas!

Q: Can we represent “If *P*, then *Q*” as a logical formula?

P	*Q*	*P*	→	*Q*
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F		F

All Logical Formulas!

Q: Can we represent “If *P*, then *Q*” as a logical formula?

P	*Q*	*P*	→	*Q*
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

All Logical Formulas!

Q: Can we represent “If *P*, then *Q*” as a logical formula?

P	*Q*	*P*	→	*Q*
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

Truth Table to Formula

Strategy:

1. Make a rule for each “True
2. OR them together

P	*Q*	*P*	→	*Q*
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

Truth Table to Formula

Strategy:

1. Make a rule for each “True
2. OR them together

AND(P,Q)

AND(NOT(P),Q)

AND(NOT(P),NOT(Q))

P	*Q*	*P*	→	*Q*
T	T	T	**T**	T
T	F	T	F	F
F	T	F	**T**	T
F	F	F	**T**	F

Truth Table to Formula

Strategy:

1. Make a rule for each “True
2. OR them together

AND(P,Q)

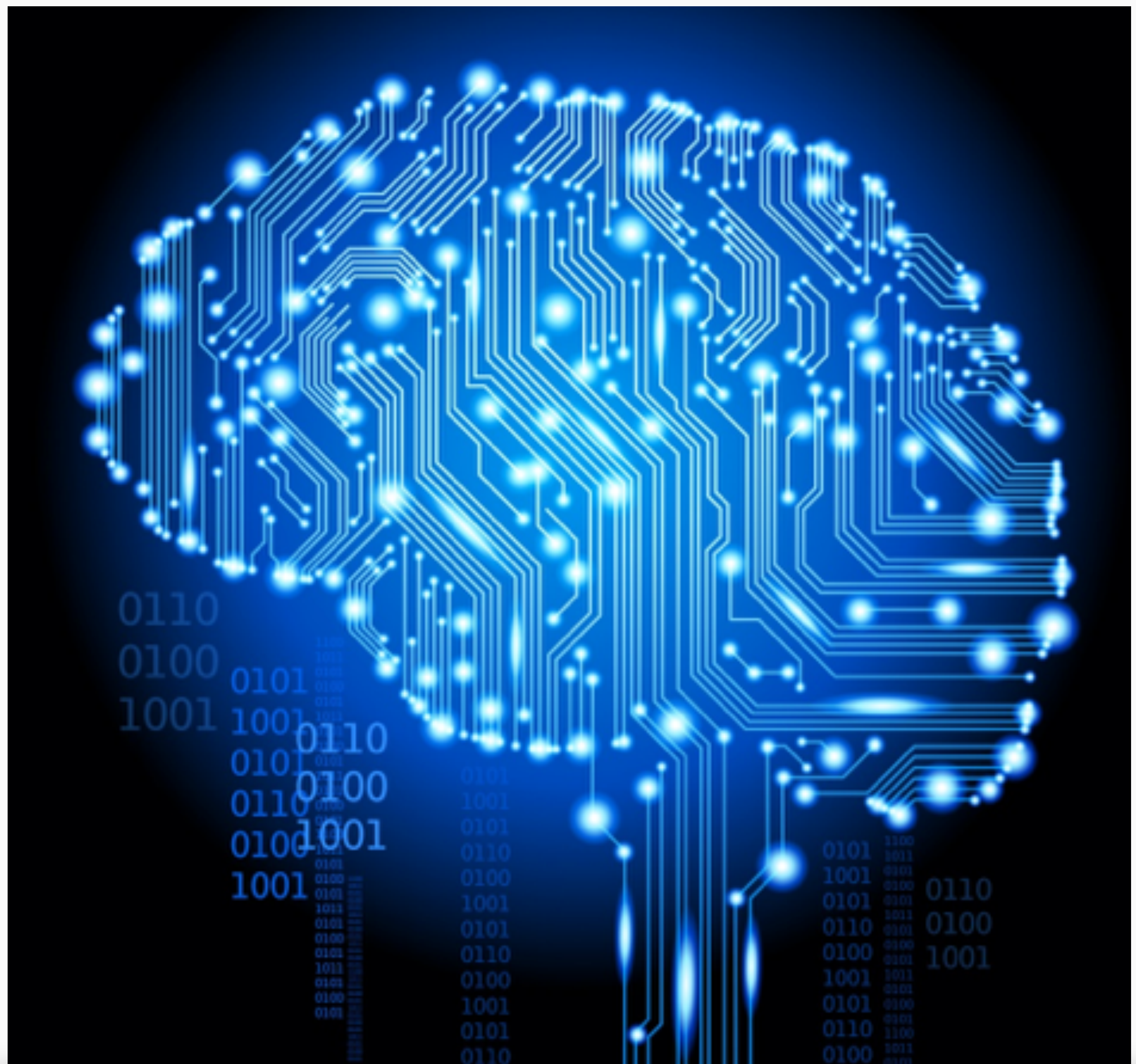
or

AND(NOT(P),Q)

or

AND(NOT(P),NOT(Q))

Logic



Onward! Gates

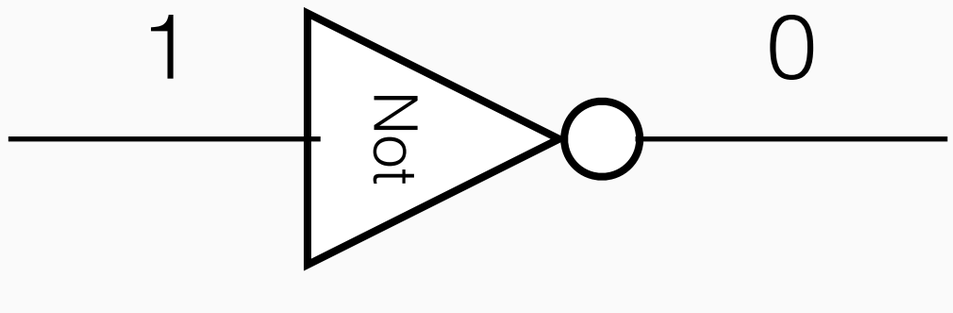
Gates: *NOT*

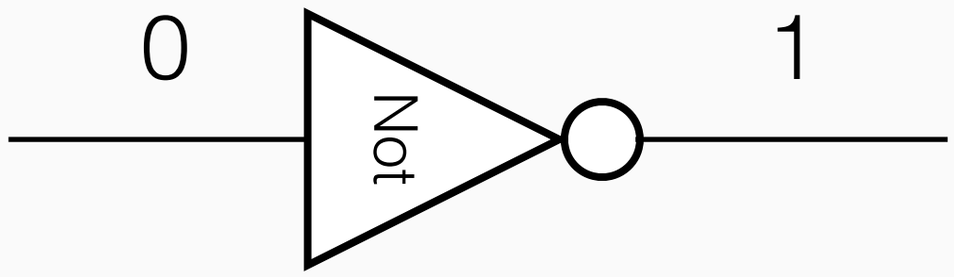
P	*NOT(P)*
T	**F**
F	**T**

P	*NOT(P)*
1	**0**
0	**1**

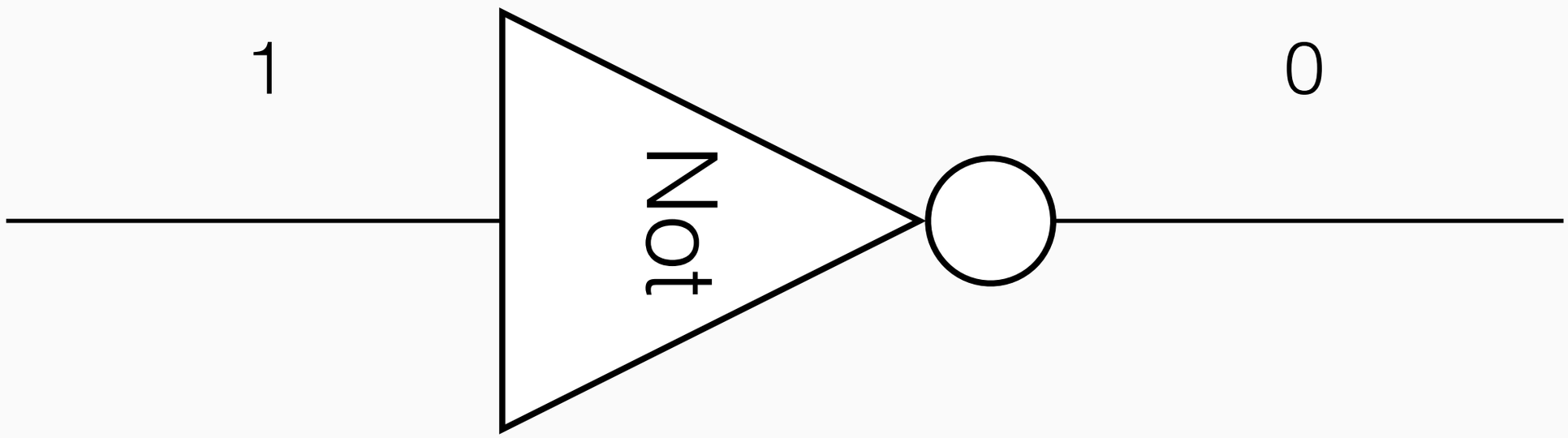
Gates: *NOT*

P	*NOT(P)*
1	**0**
0	**1**

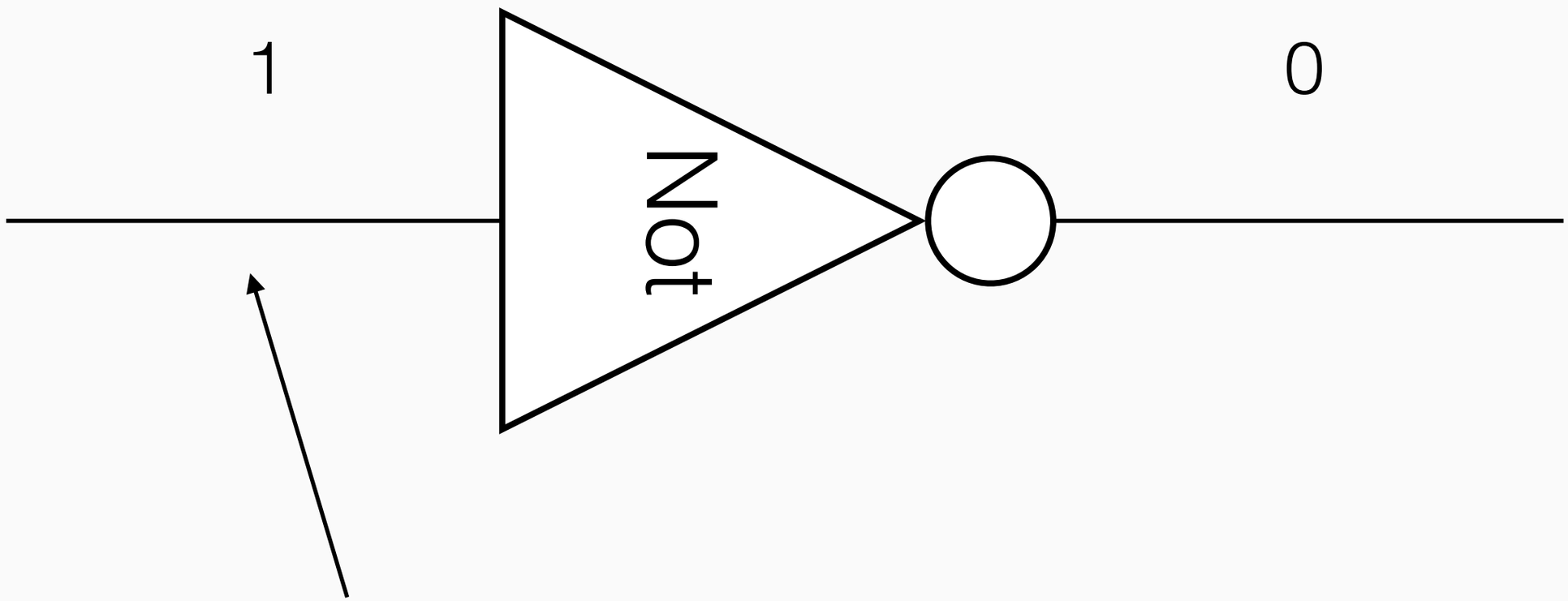




Gates: *NOT*

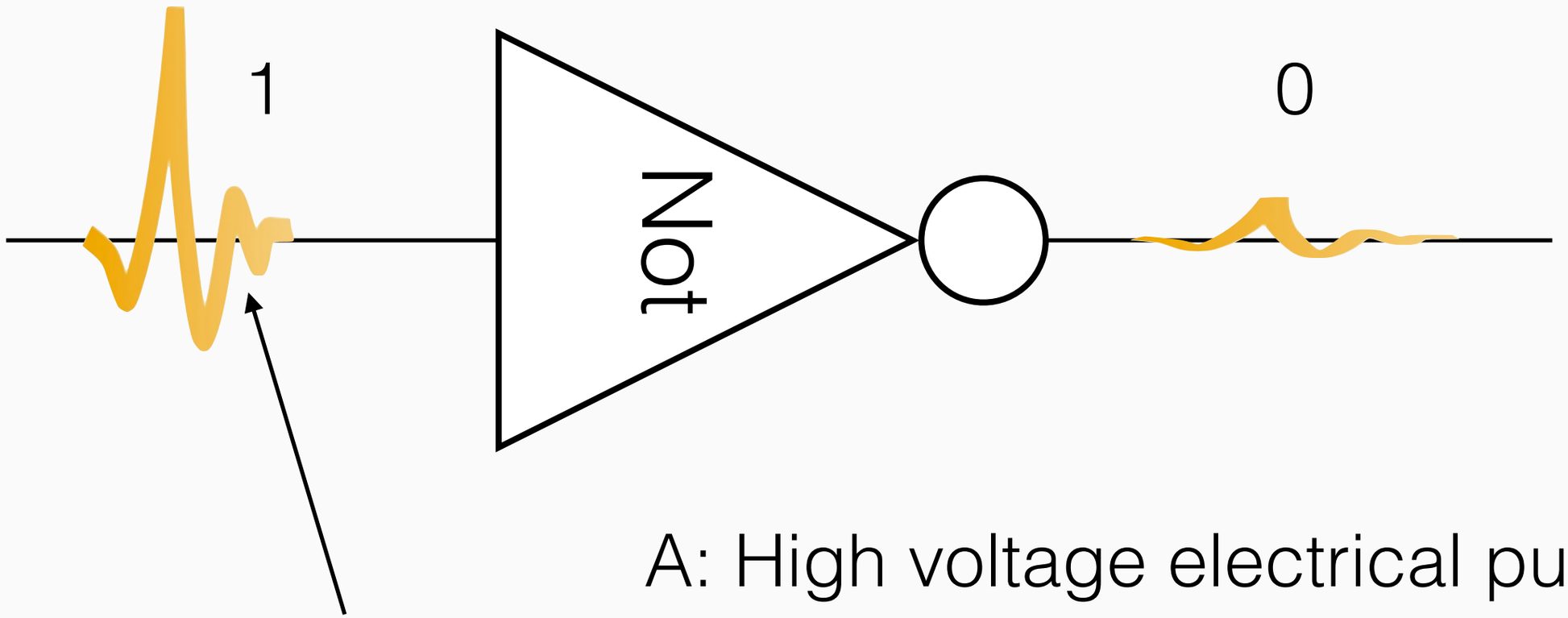


Gates: *NOT*



Q: What is this, physically?

Gates: *NOT*

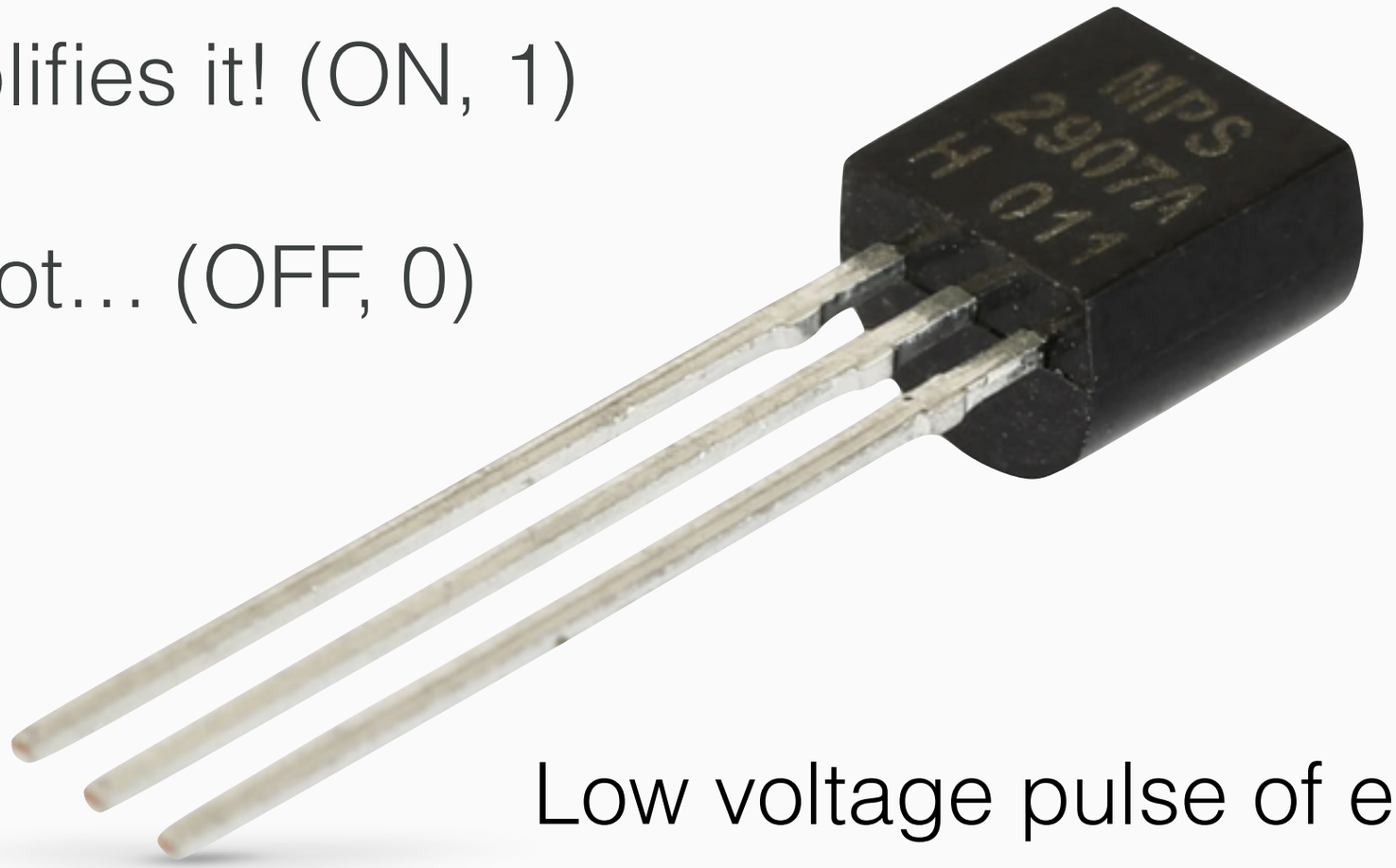


Q: What is this, physically?

A: High voltage electrical pulses!

Now: The Transistor

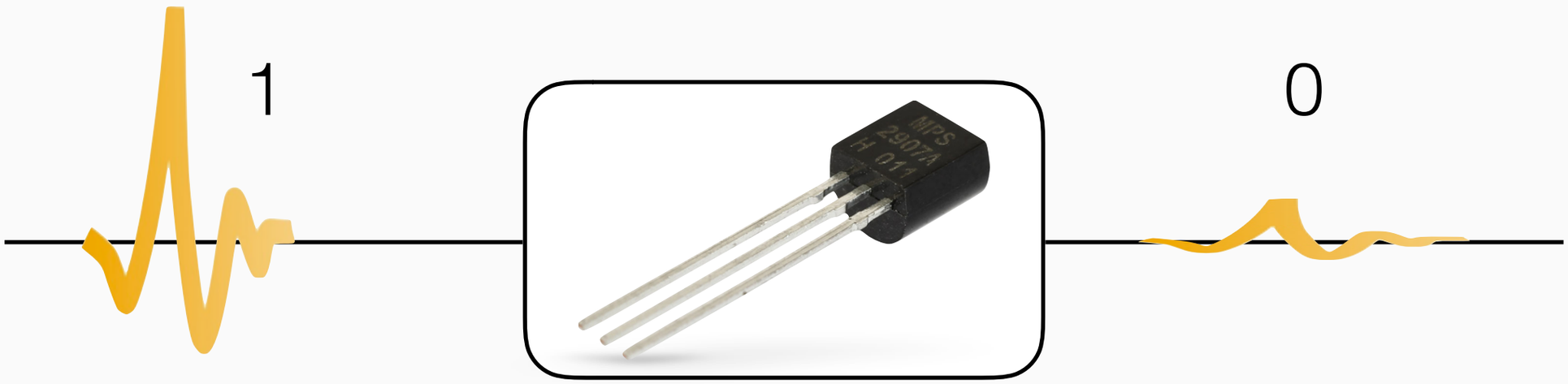
- Takes in electric current:
  - Amplifies it! (ON, 1)
  - Or not… (OFF, 0)



Low voltage pulse of electricity = 0

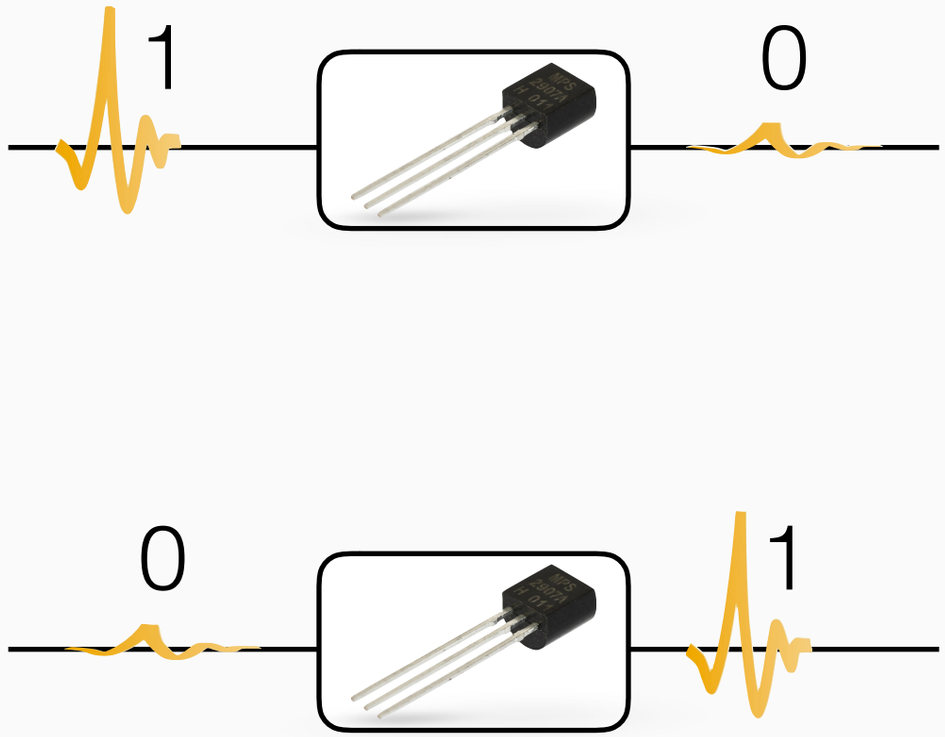
High voltage pulse of electricity = 1

Gates: *NOT*



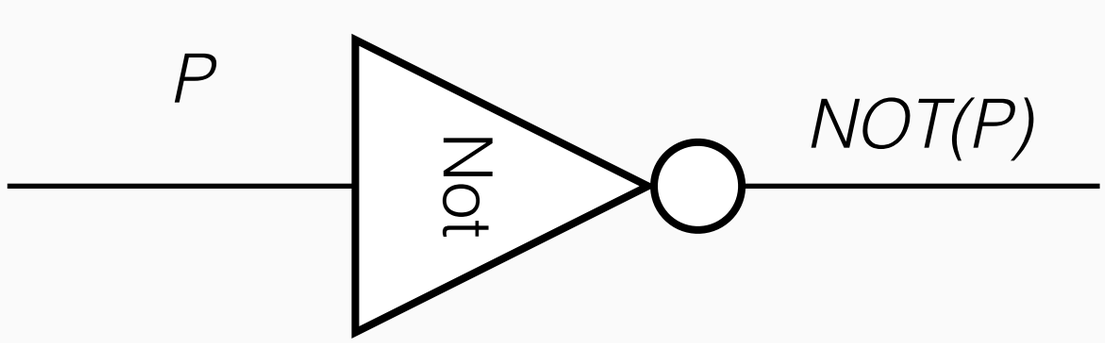
Gates: *NOT*

P	*NOT(P)*
1	**0**
0	**1**



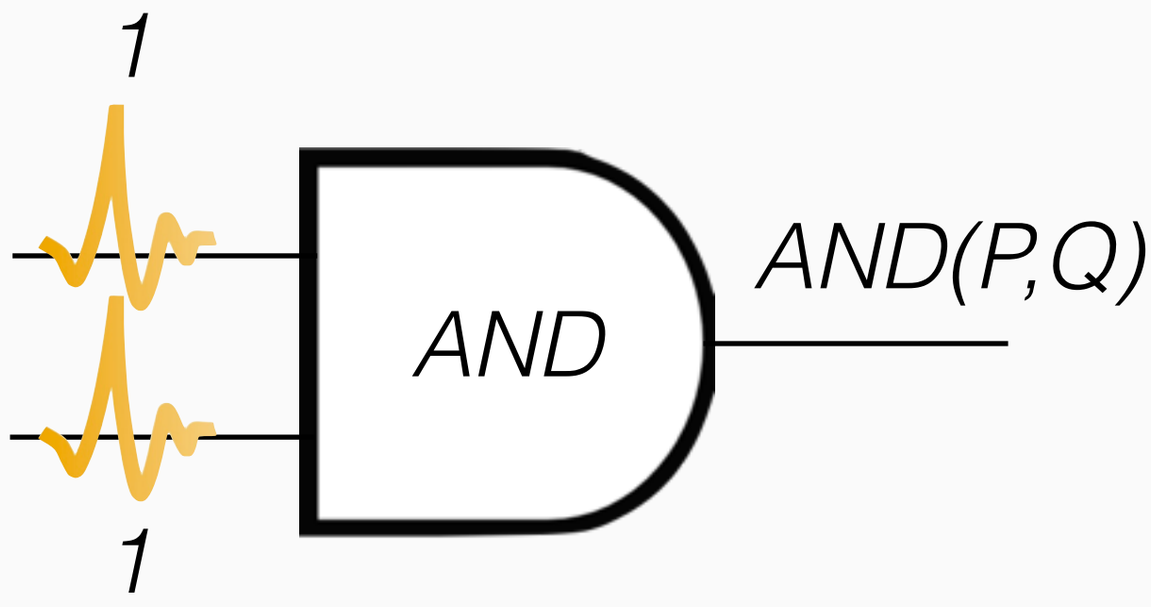
Gates: *NOT*

P	*NOT(P)*
1	**0**
0	**1**



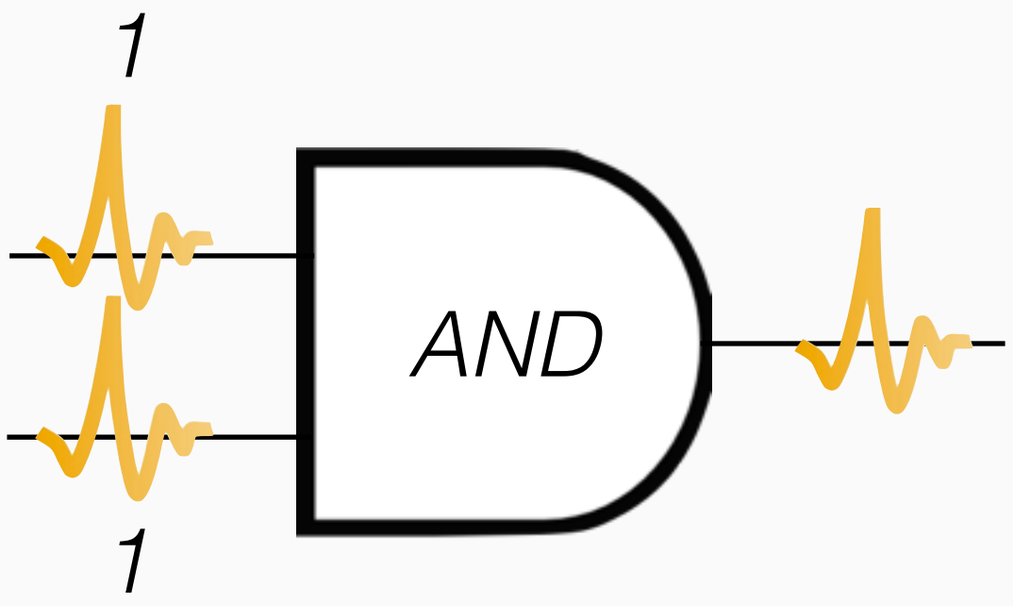
Gates: *AND*

P	*Q*	*AND(*	*P*	*Q)*
1	1	**1**	1	1
1	0	**0**	1	0
0	1	**0**	0	1
0	0	**0**	0	0



Gates: *AND*

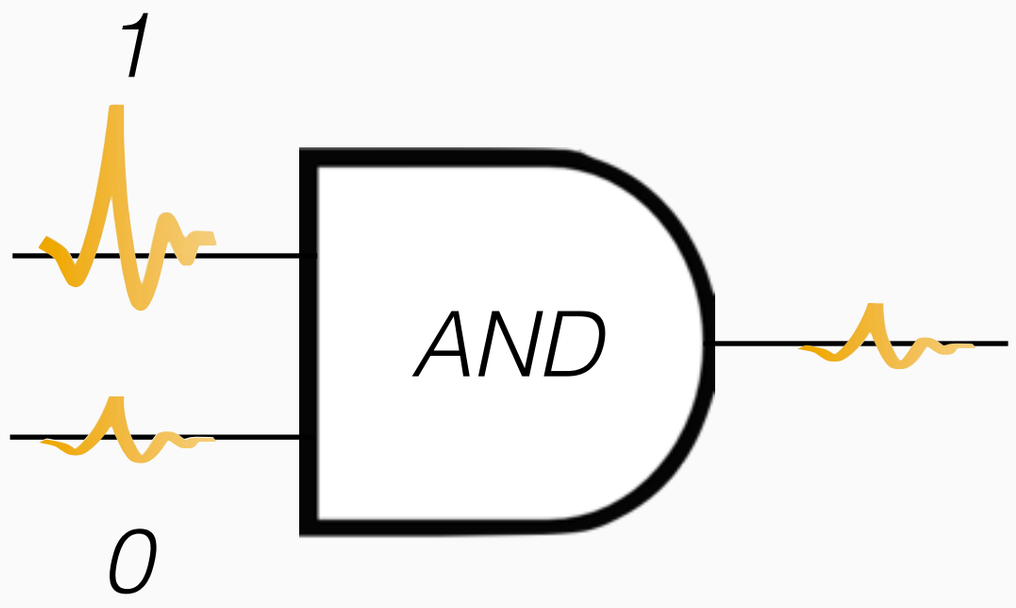
P	*Q*	*AND(*	*P*	*Q)*
1	1	**1**	1	1
1	0	**0**	1	0
0	1	**0**	0	1
0	0	**0**	0	0



AND(P,Q) = 1

Gates: *AND*

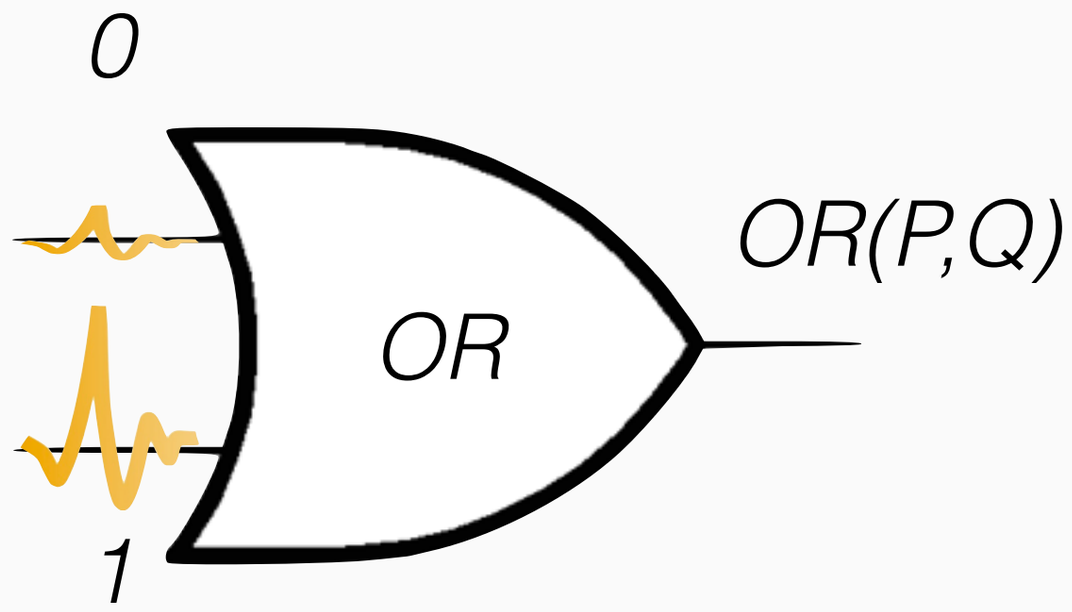
P	*Q*	*AND(*	*P*	*Q)*
1	1	**1**	1	1
1	0	**0**	1	0
0	1	**0**	0	1
0	0	**0**	0	0



AND(P,Q) = 0

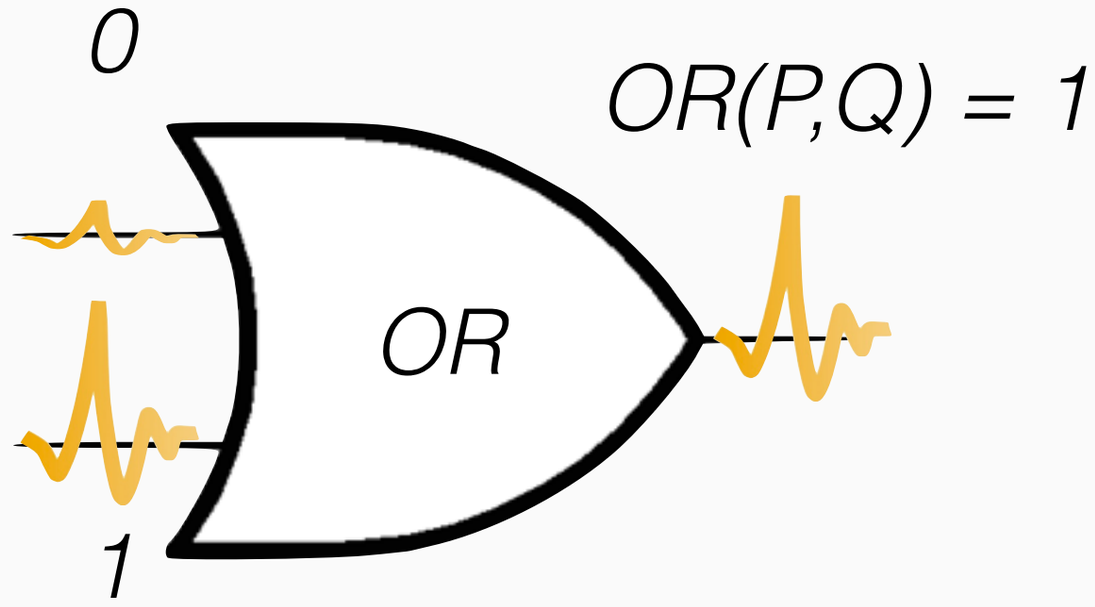
Gates: *OR*

P	*Q*	*OR(*	*P*	*Q)*
1	1	**1**	1	1
1	0	**1**	1	0
0	1	**1**	0	1
0	0	**0**	0	0



Gates: *OR*

P	*Q*	*OR(*	*P*	*Q)*
1	1	**1**	1	1
1	0	**1**	1	0
0	1	**1**	0	1
0	0	**0**	0	0



Gates: Composition

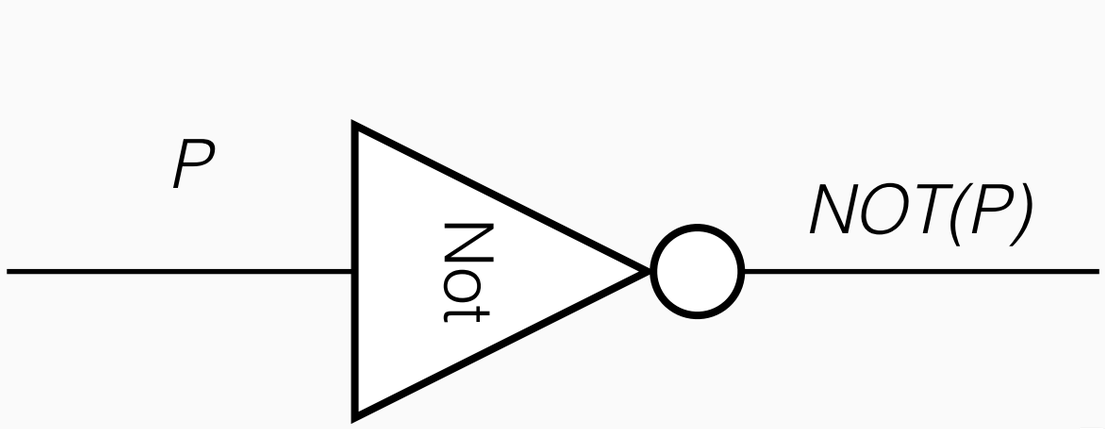
Try writing down the gate structure for the following Boolean sentence:

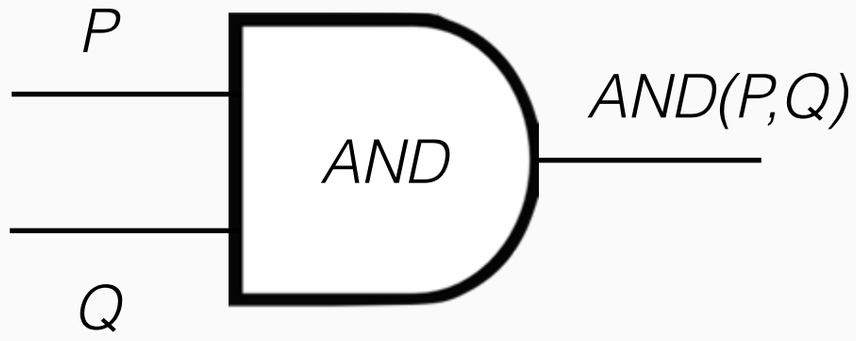
OR(P,NOT(Q))

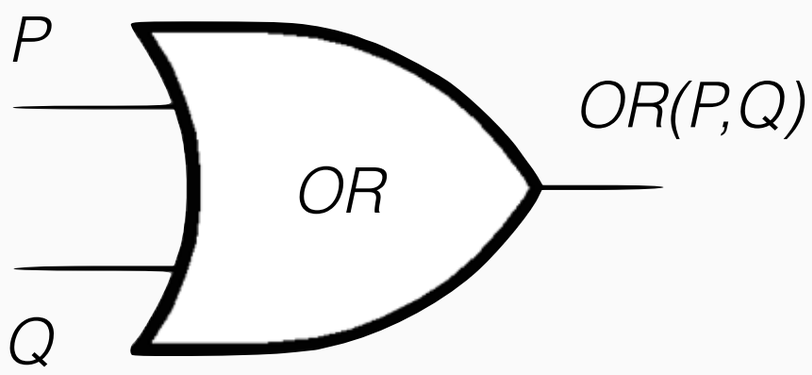
Gates: Composition

Try writing down the gate structure for the following Boolean sentence:

OR(P,NOT(Q))





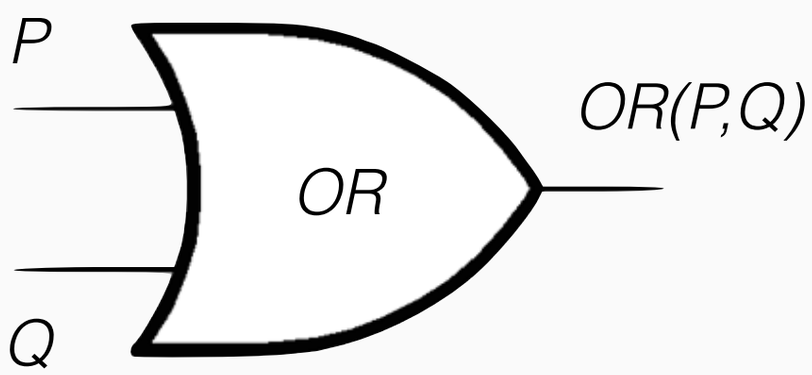


Gates: Composition

Try writing down the gate structure for the following Boolean sentence:

OR(P,NOT(Q))

A:



Gates: Composition

Try writing down the gate structure for the following Boolean sentence:

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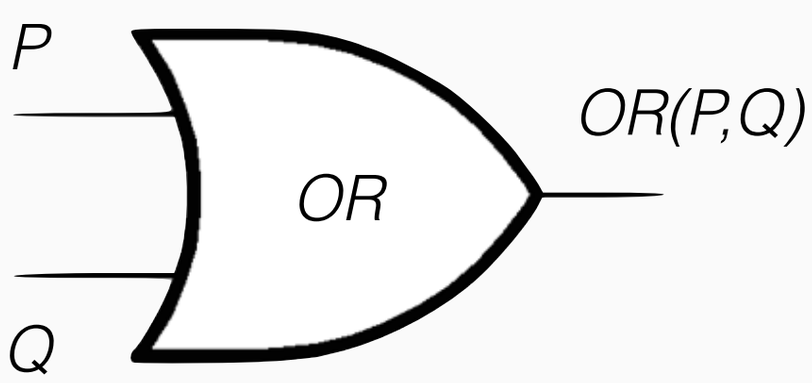
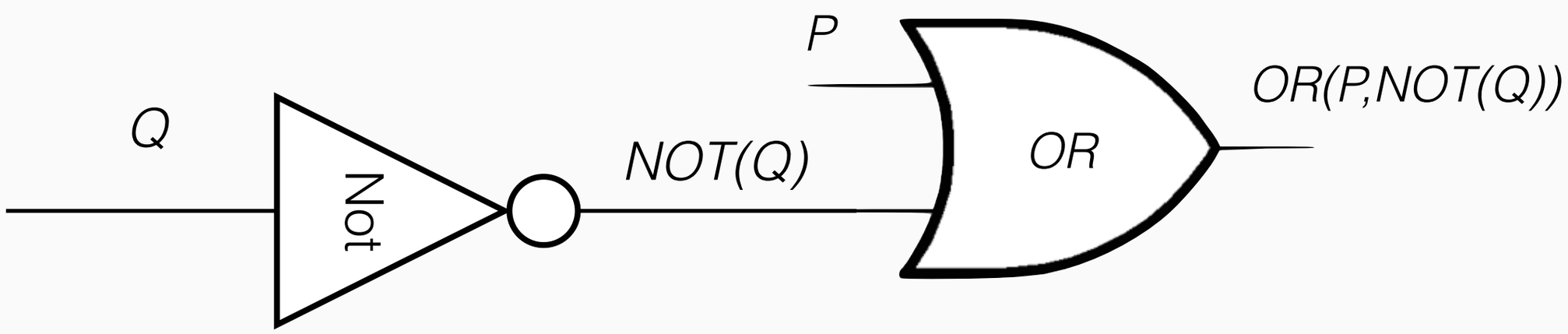


Gates: Composition

Try writing down the gate structure for the following Boolean sentence:

OR(P,NOT(Q))

A:

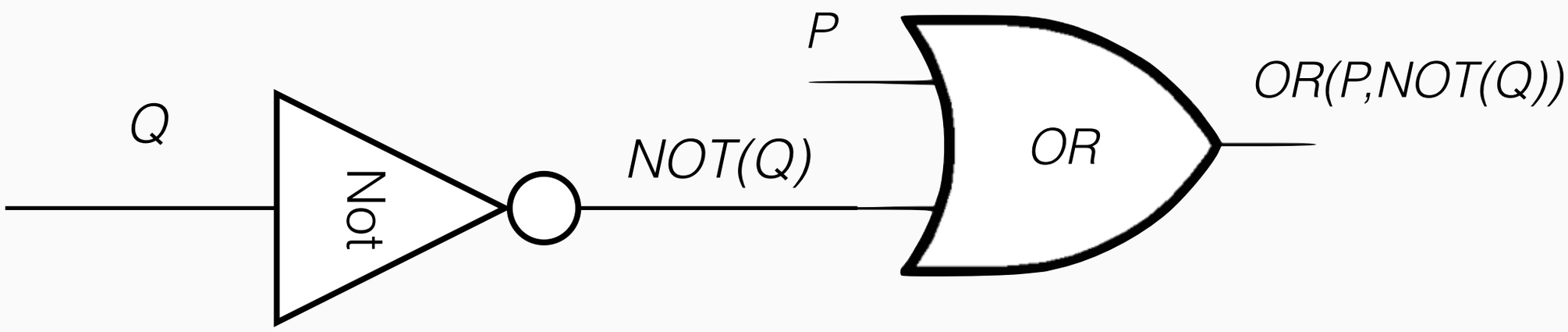


Gates: Composition

OR(P,NOT(Q))

P	*Q*	*OR(*	*P*	*NOT(*	*Q)*
T	T	**T**	T	F	T
T	F	**T**	T	T	F
F	T	**F**	F	F	T
F	F	**T**	F	T	F

A:

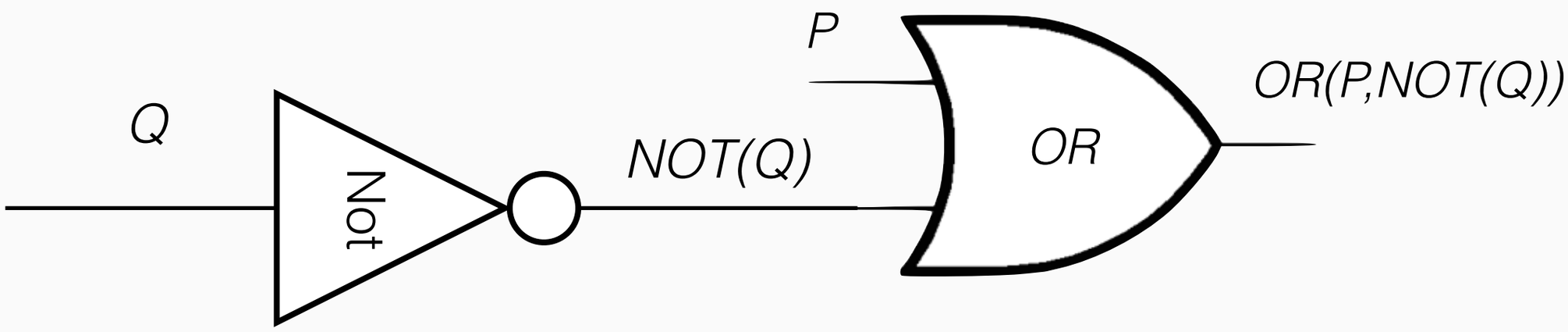


Gates: Composition

OR(P,NOT(Q))

P	*Q*	*OR(*	*P*	*NOT(*	Q)
T	T	**T**	T	F	T
T	F	**T**	T	T	F
F	T	**F**	F	F	T
F	F	**T**	F	T	F

A:

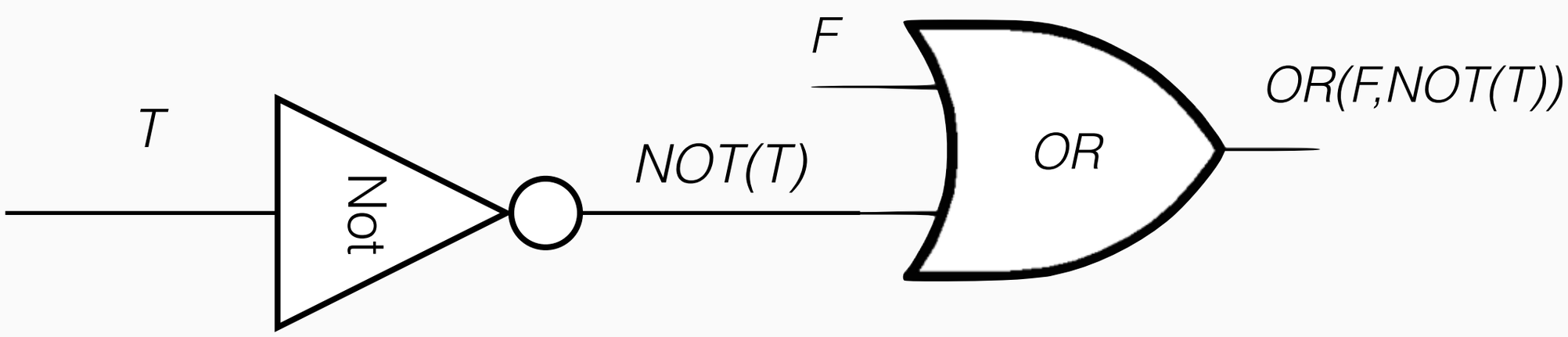


Gates: Composition

OR(P,NOT(Q))

P	*Q*	*OR(*	*P*	*NOT(*	Q)
T	T	**T**	T	F	T
T	F	**T**	T	T	F
F	T	**F**	F	F	T
F	F	**T**	F	T	F

A:

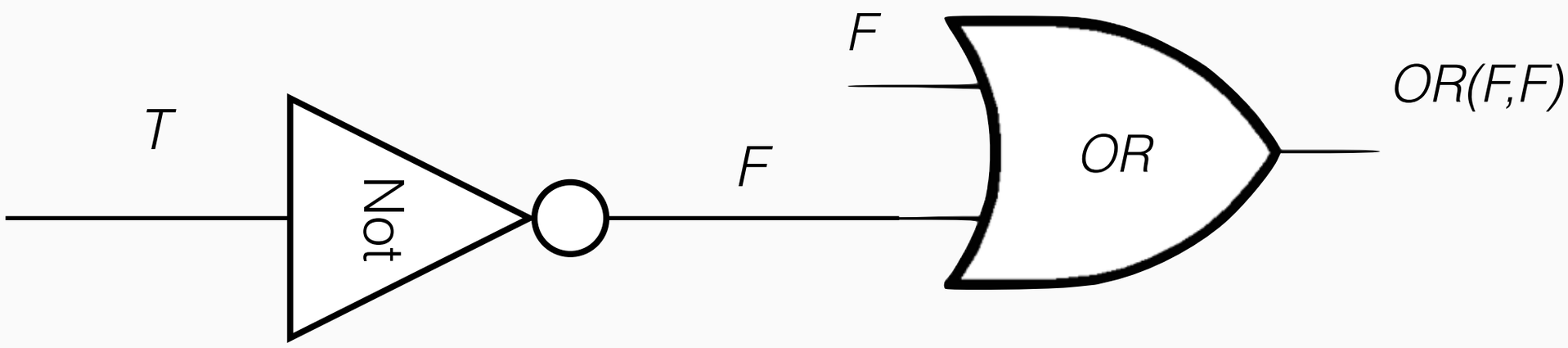


Gates: Composition

OR(P,NOT(Q))

P	*Q*	*OR(*	*P*	*NOT(*	Q)
T	T	**T**	T	F	T
T	F	**T**	T	T	F
F	T	**F**	F	F	T
F	F	**T**	F	T	F

A:

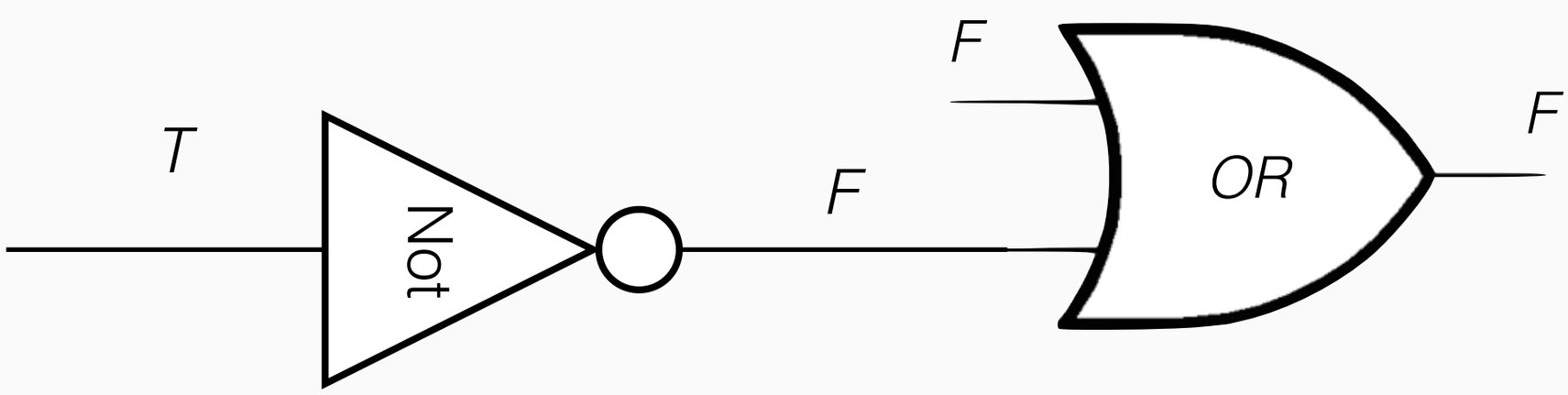


Gates: Composition

OR(P,NOT(Q))

P	*Q*	*OR(*	*P*	*NOT(*	Q)
T	T	**T**	T	F	T
T	F	**T**	T	T	F
F	T	**F**	F	F	T
F	F	**T**	F	T	F

A:

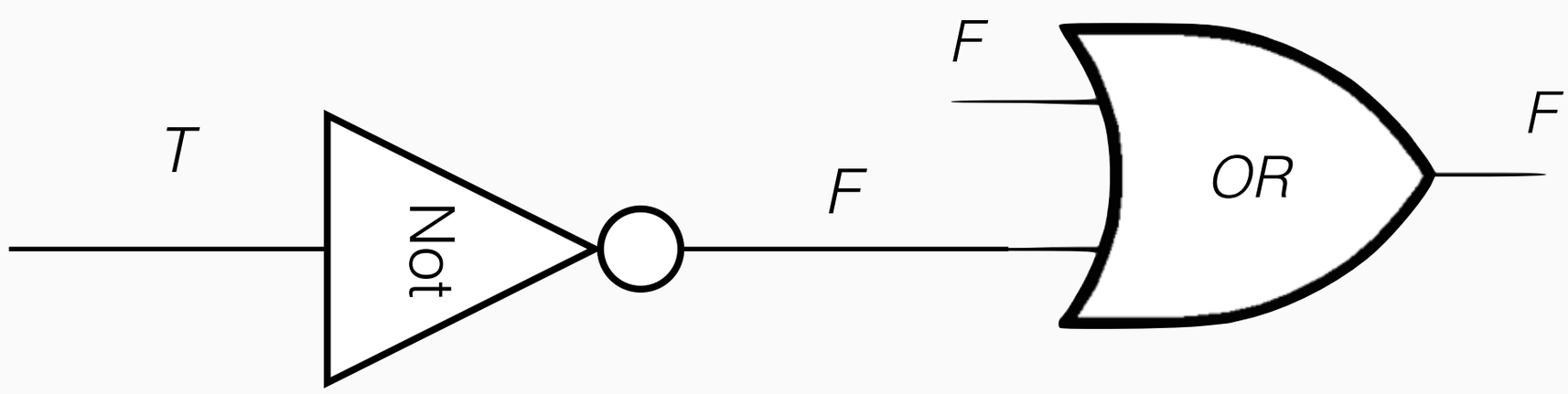


Gates: Composition

OR(P,NOT(Q))

P	*Q*	*OR(*	*P*	*NOT(*	Q)
T	T	**T**	T	F	T
T	F	**T**	T	T	F
F	T	**F**	F	F	T
F	F	**T**	F	T	F

A:



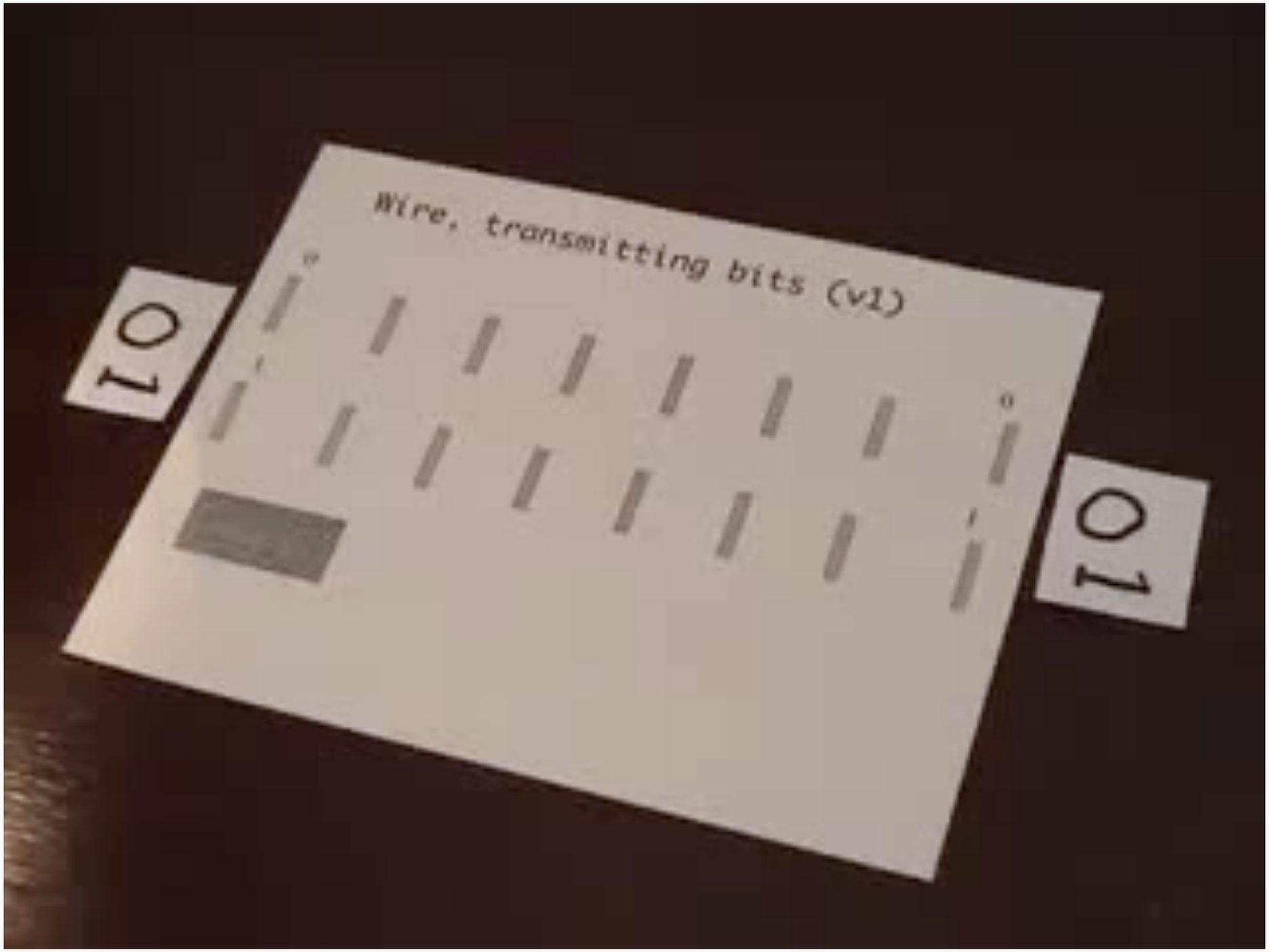
Truth Table to Formula

Idea: with a certain set of logical functions, we can represent all possible logical formulas!

Truth Table to Gate

Idea: with a certain set of logical ~~functions~~ gates, we can represent all possible logical formulas!

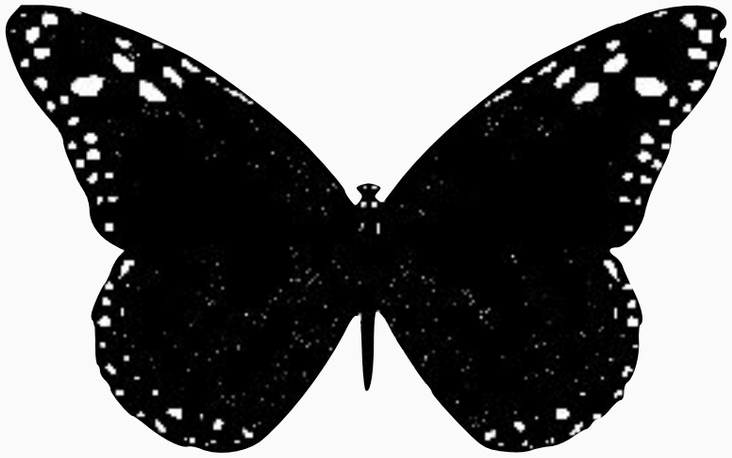
What Else Could be a Gate?

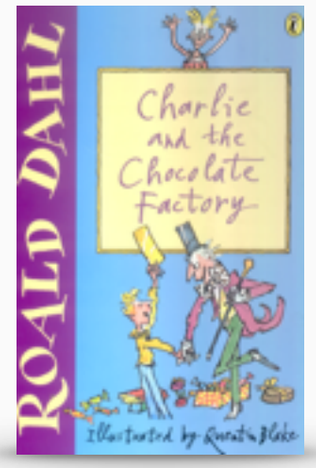


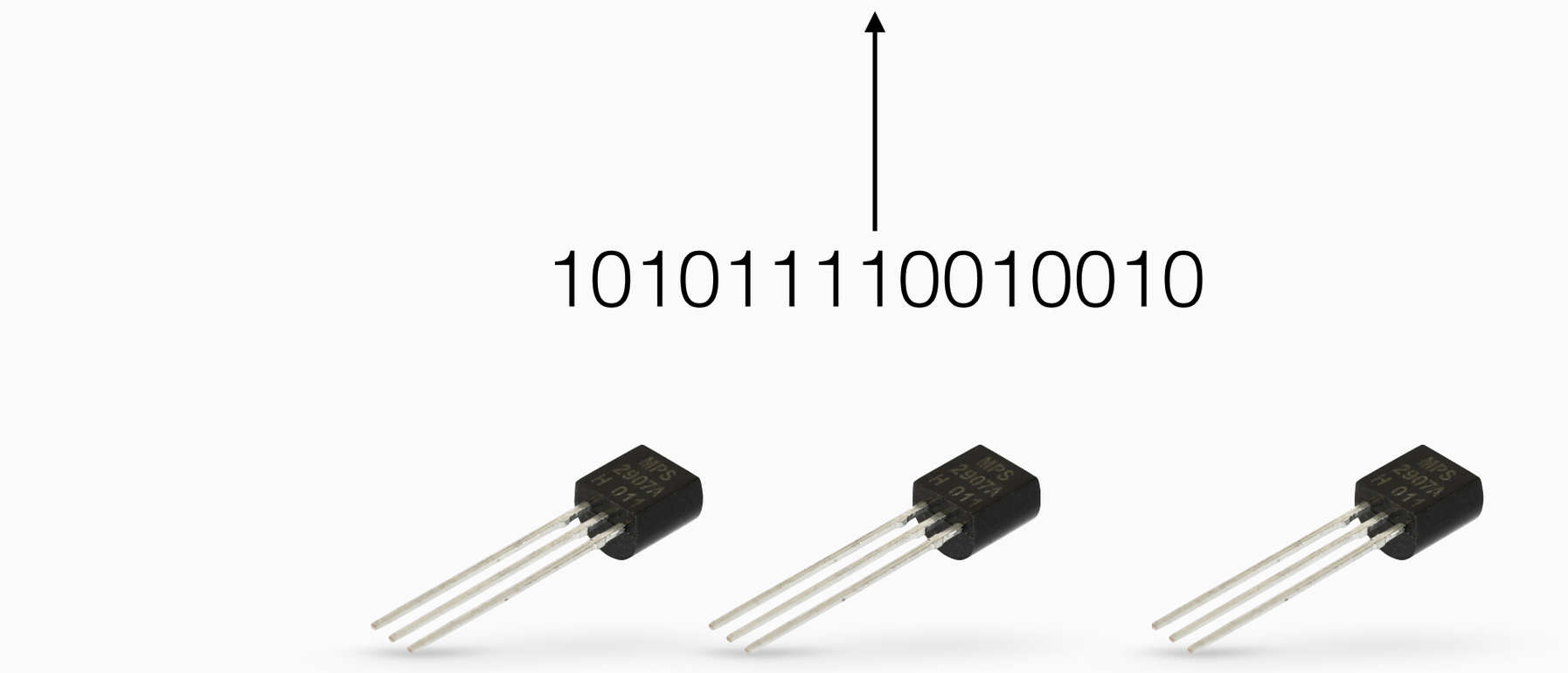
Could It Work?

- Michael's domino *OR* gate: 24 dominoes
- The first pentium processor had 3.3 Mill transistors, or roughly 800k gates.
- So we need around 20 Mill dominoes
- World record for domino topple: 4.5 Mill
- Pentium: computes 60 Mill times a second
- Dominoes? Takes awhile to set up…

Abstraction!



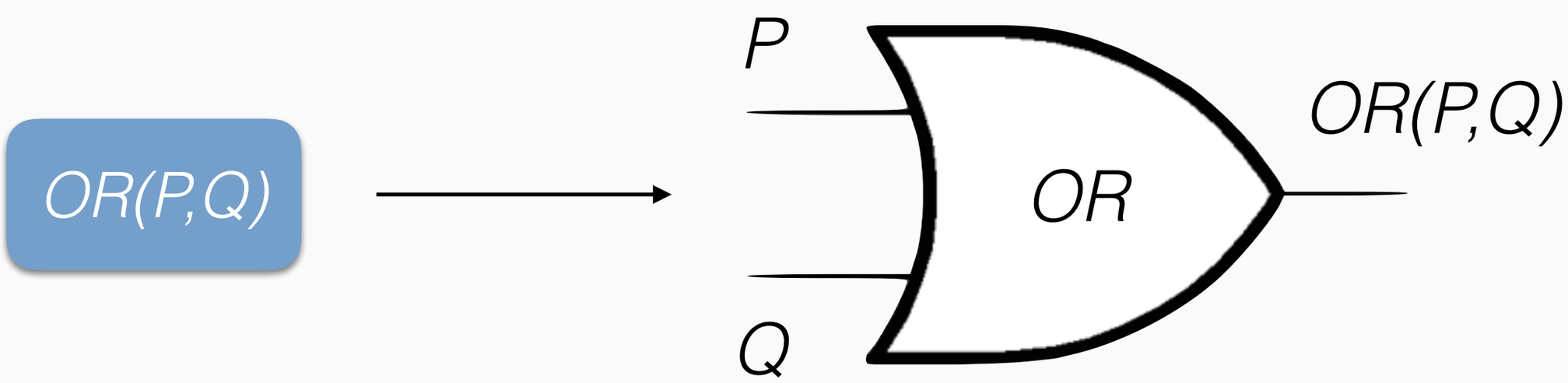




Abstraction!

OR(P,Q)

Abstraction!



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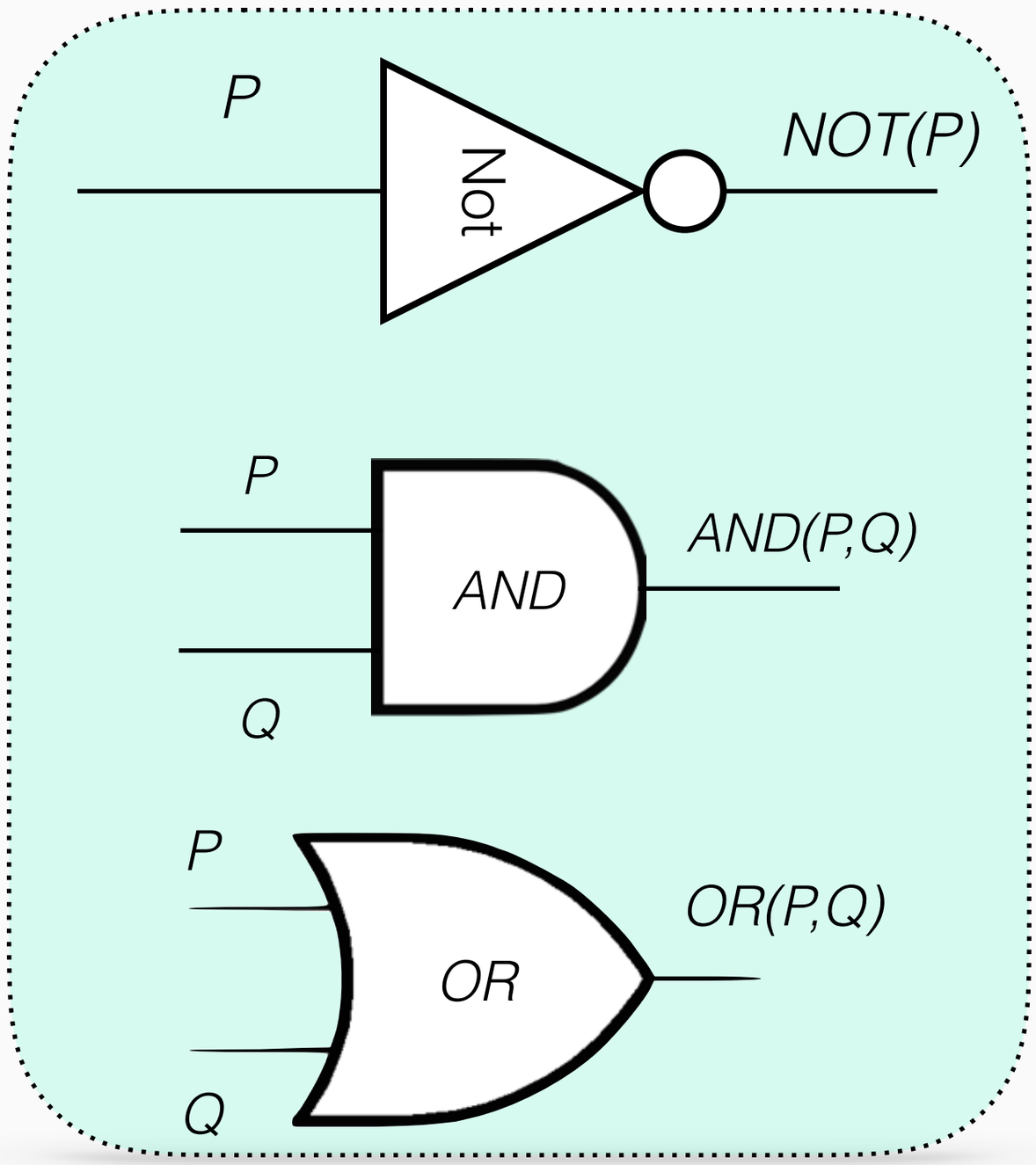
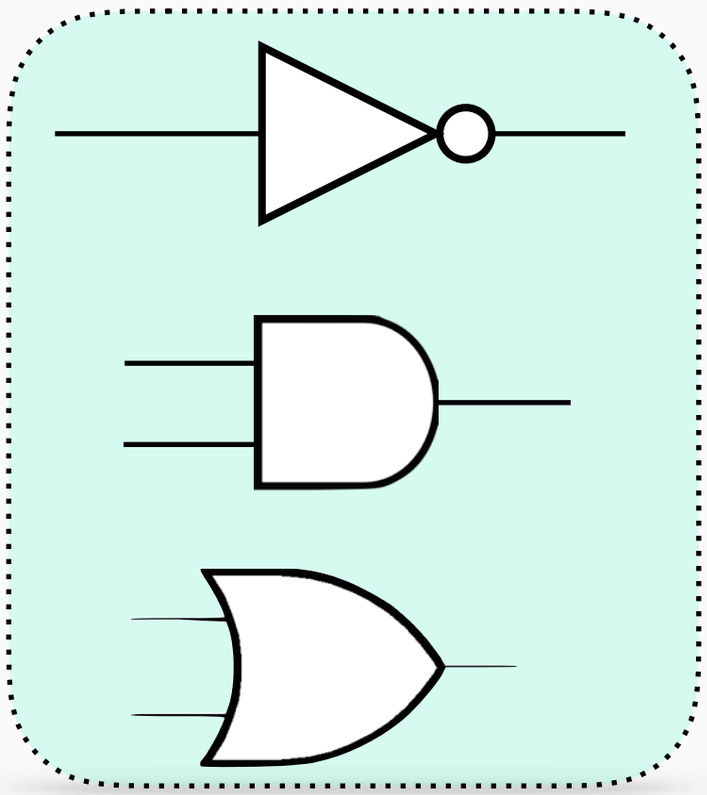


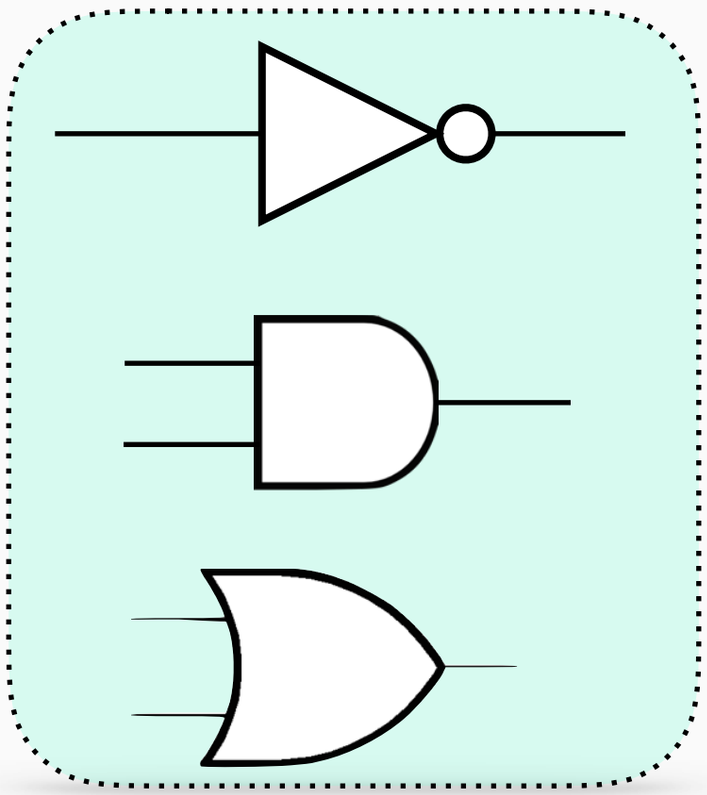
all possible logical formulas!

Abstraction:



Low level programs

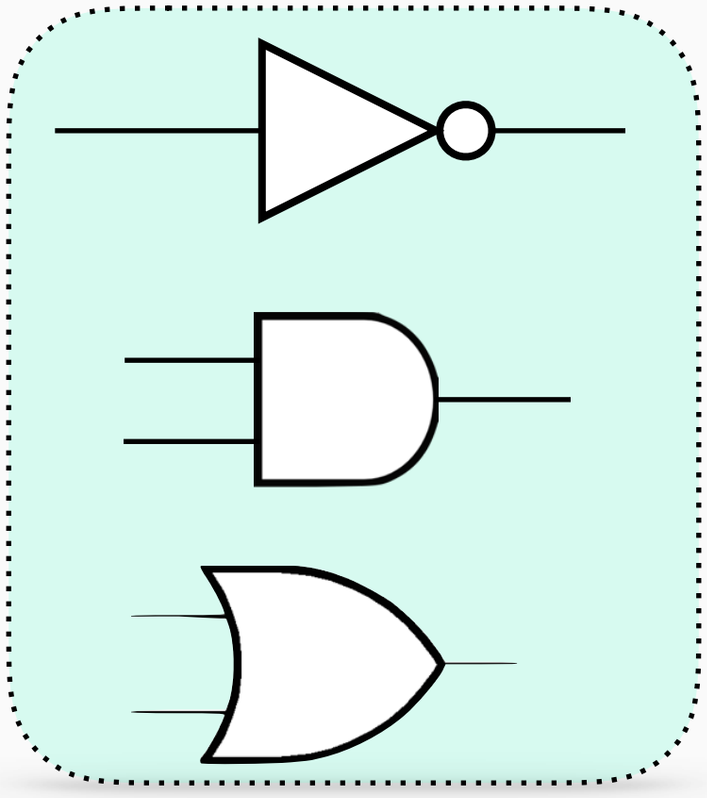
Abstraction:



Low level programs

High level programs

Abstraction:



Low level programs

High level programs

=

Your ideas!

Reflection

- Logic review
- Gates
  - *AND, OR, NOT* gates
  - Composition of gates
  - Can represent all possible logical formulas as gates
  - Transistors and friends are just gates
- **Up Next:** Turning gates into simple programs!