Unit 7: Theory

Dave Abel

March 12th, 2016

As Promised:

Any Midterm questions?

Also: one side of a 3”x 5” index card

AI Update!

AlphaGo 3-1 against Lee Sedol

(He did it! Woohoo!)

Theory

- Revisiting Growth Rates
- Problem Classes
  - *SOLVE*
  - *VERIFY*
- The biggest unanswered question in Computer Science!
  - Implications
- Unsolvable problems
- Uncountable things
- Measuring Simplicity, Occam's Razor

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Theory: Takeaway

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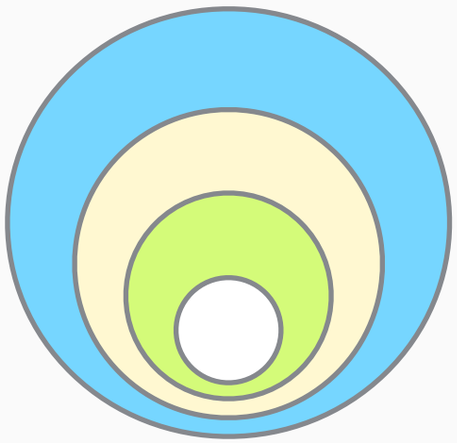

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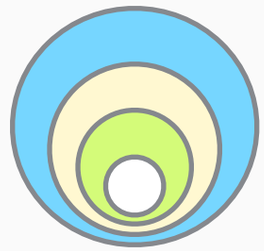

Q: Are problems in VERIFY, also in SOLVE?

This is the biggest unanswered question in computer science.

Growth Rate: Definition

1. **Definition:** The *growth rate* of an algorithm is the number of primitive operations an algorithm must execute, in the worst case, in order to complete its job.

2. We call it the *growth rate* because it's how the number of operations the computer has to execute *grows* as the size of our input grows.

I.e. sort a length 2 list vs. sorting a length 203487 list

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Reminder: *In the worst case!*

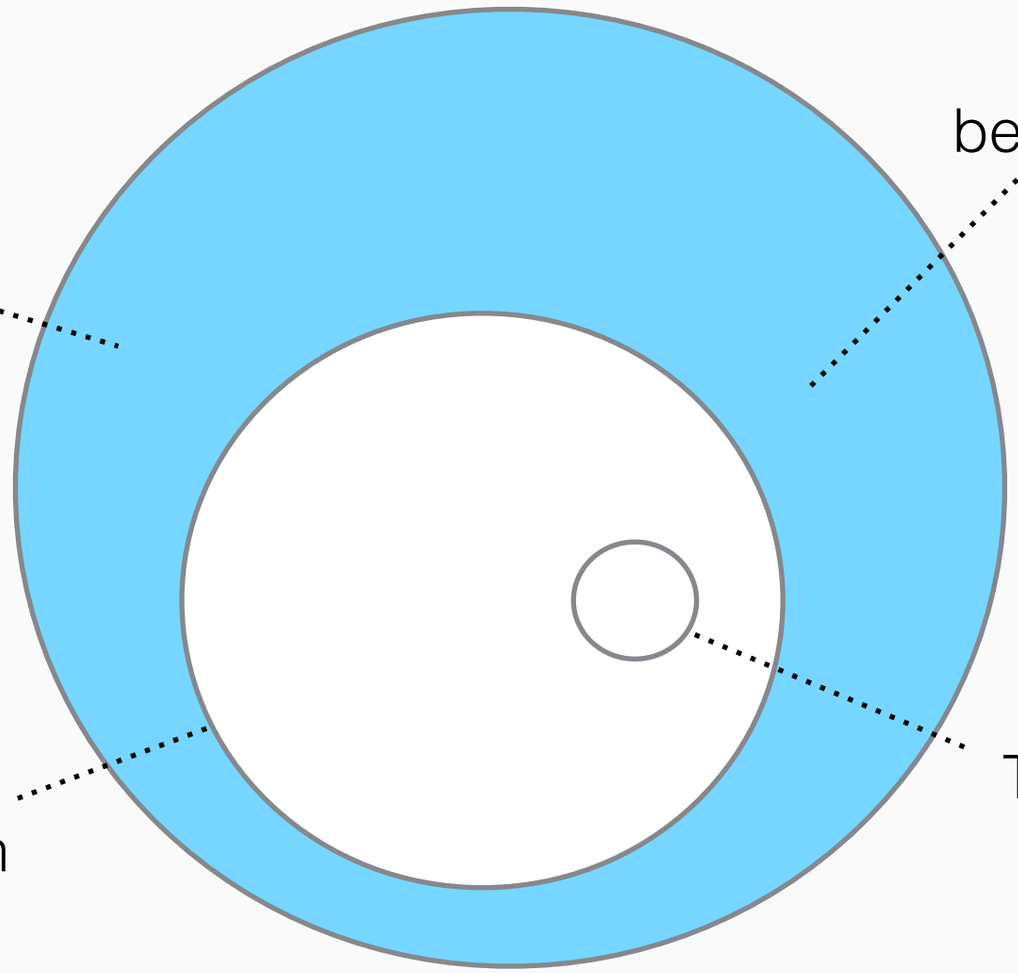
Revisiting Growth Rates

Remember Random Search?

It took *way* longer with a longer list.

Revisiting Growth Rates

Q: What, if anything, is out here?



Things that can be computed, period.

Things a regular computer can compute before the sun goes supernova

Things a domino computer could compute before the sun goes supernova

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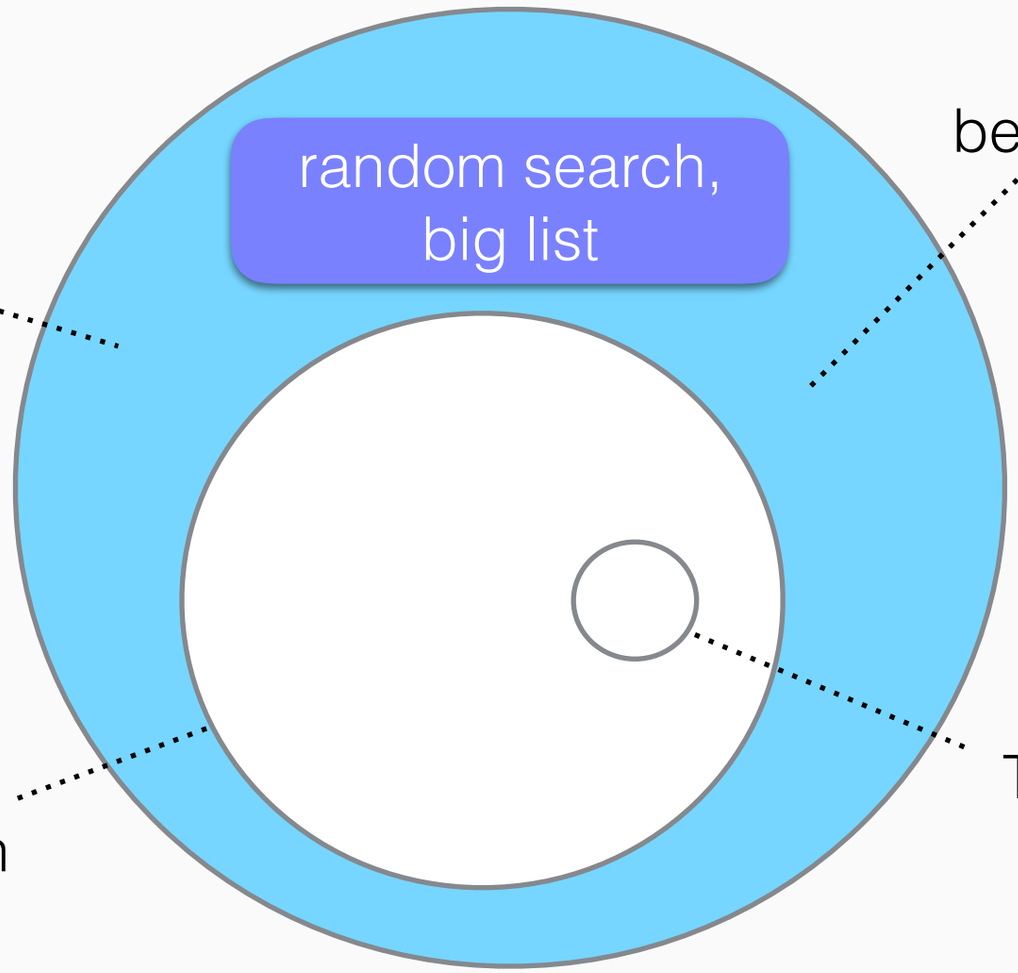

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Problem Specification Example

- *INPUT: Map of solar system, description of physical laws, summary of current technology.*
- *OUTPUT: A method for colonizing Mars.*

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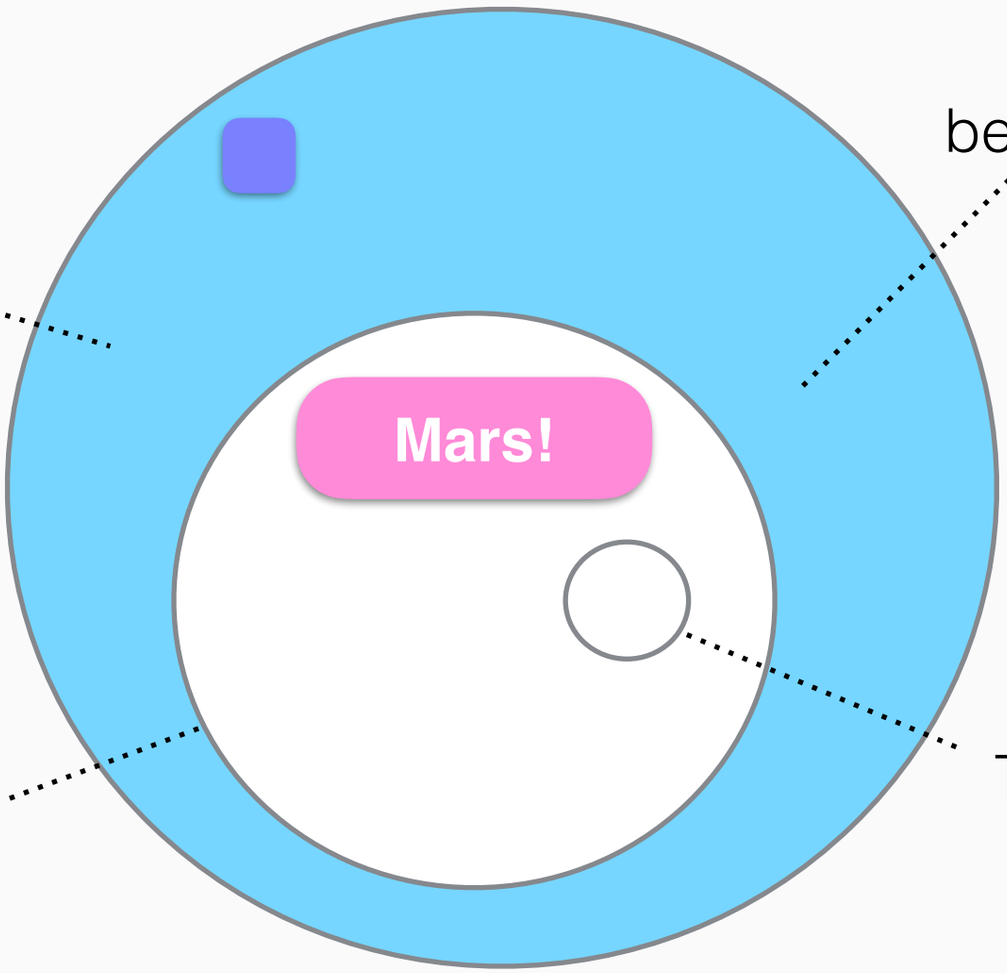

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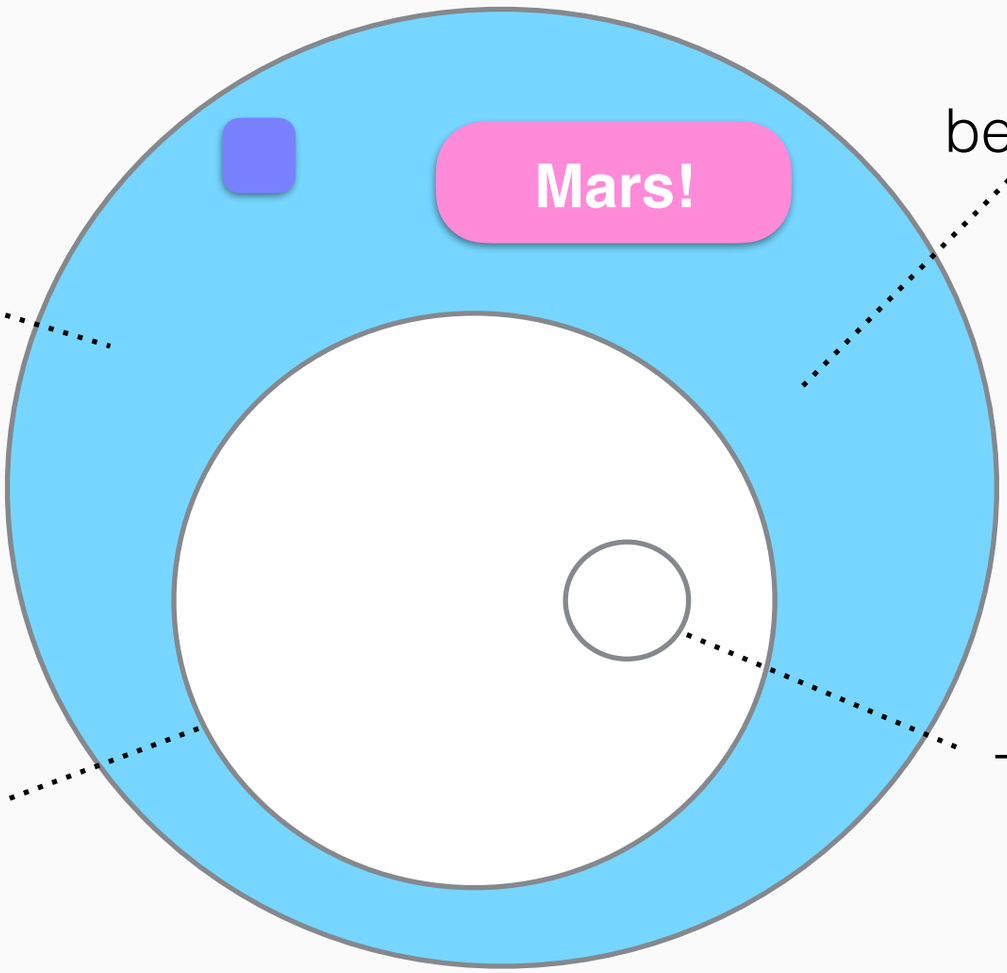

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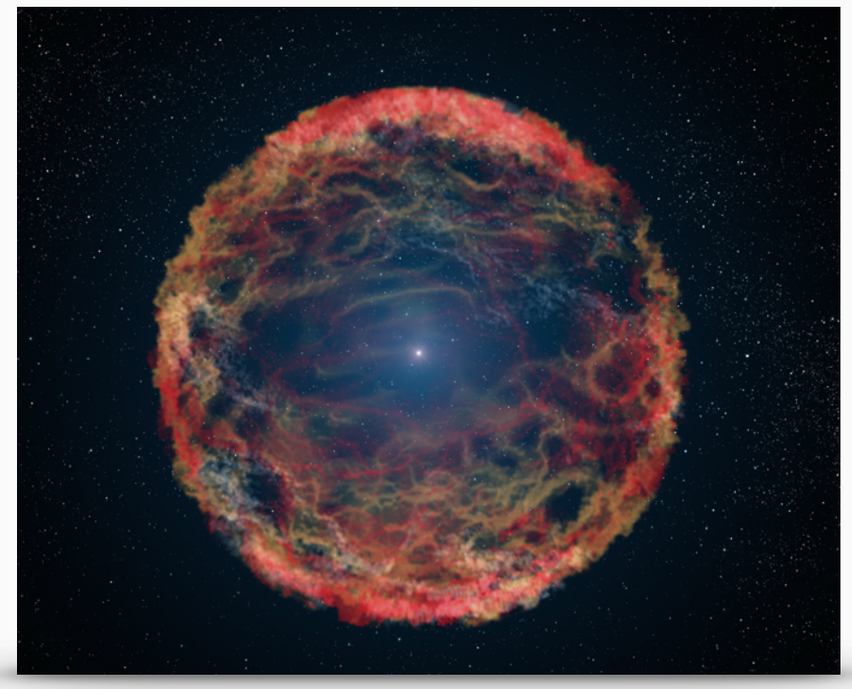
Growth Rates: The Point



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The Point: we want to know how many things we have to do as our input grows, because we want to know *what problems are solvable before the sun goes poof! (and which ones will take the drop of a hat)*

Growth Rates

- So far, we've talked about the growth rates of *algorithms*, e.g. Binary Search, Selection Sort:
  - Linear Search:

Growth Rates

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- **But don't we care about *problems*?**

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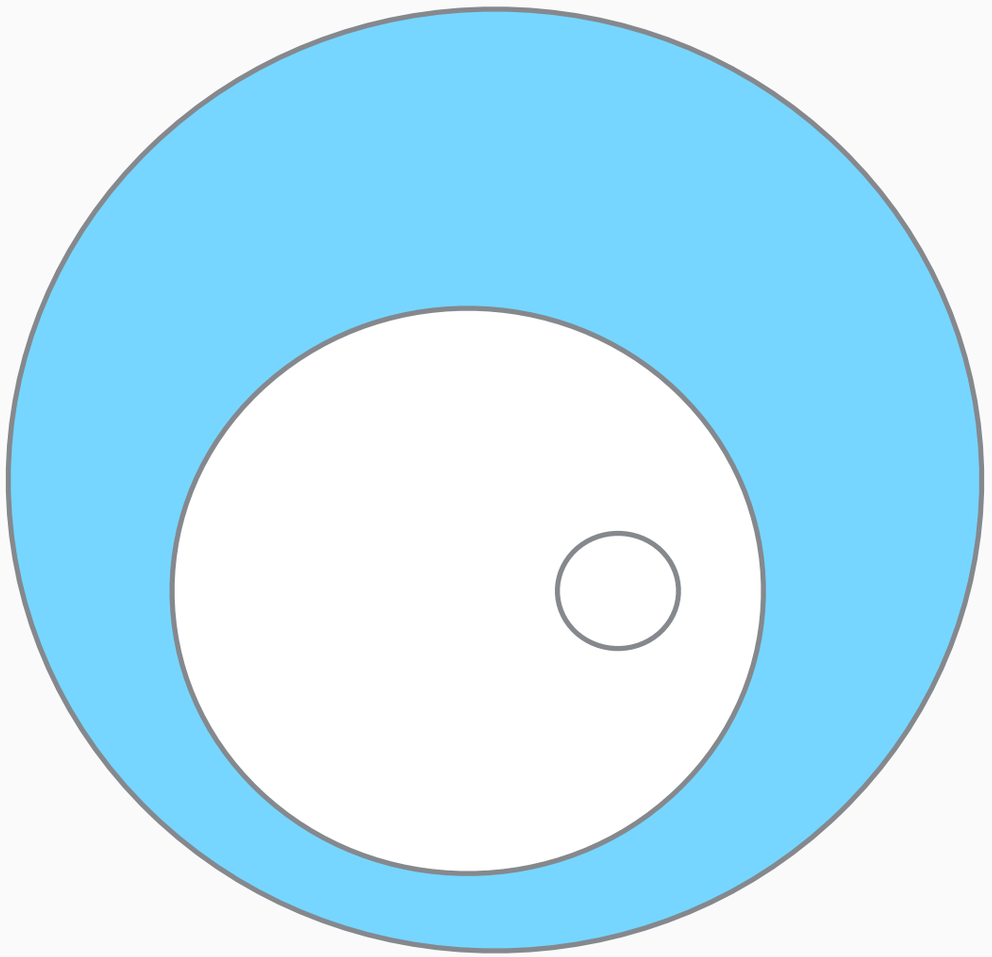
Q: For a given problem, what's the *fastest possible algorithm for solving that problem?*

Q: Moreover, what's the *growth rate* of the fastest (correct) algorithm for solving each problem?

Growth Rates



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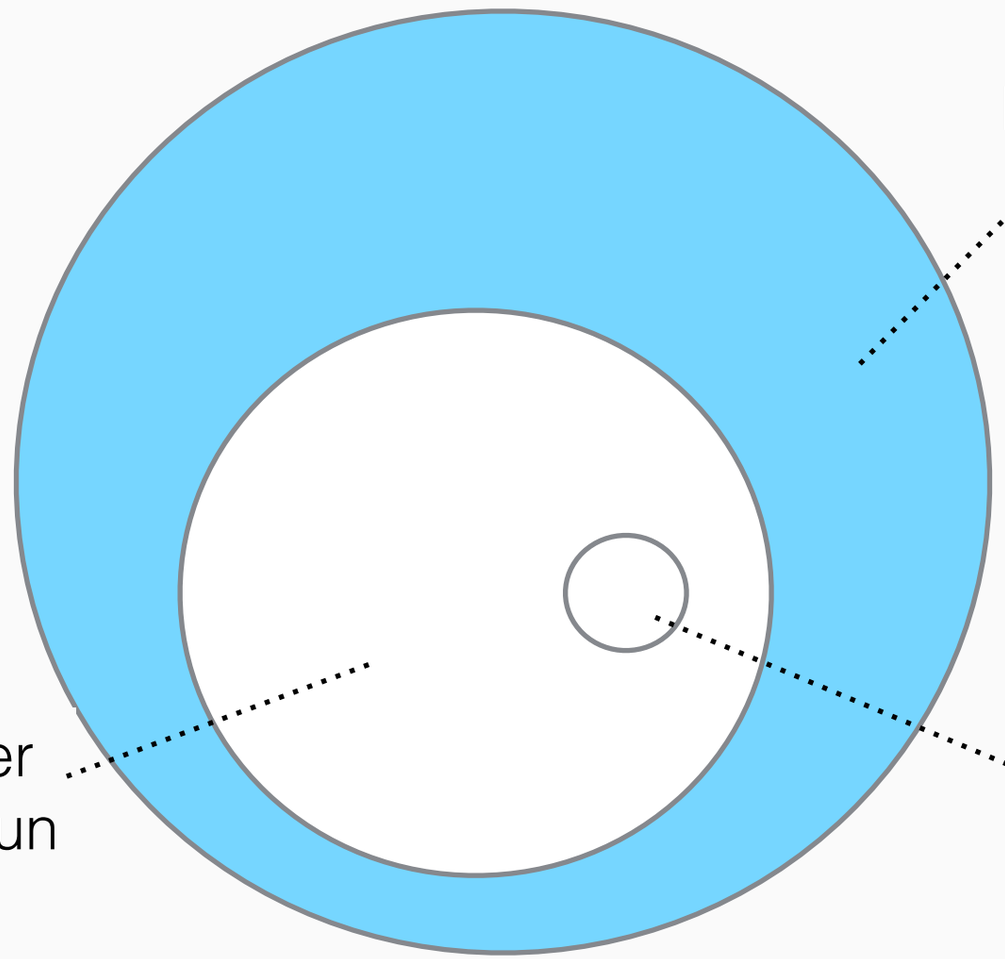

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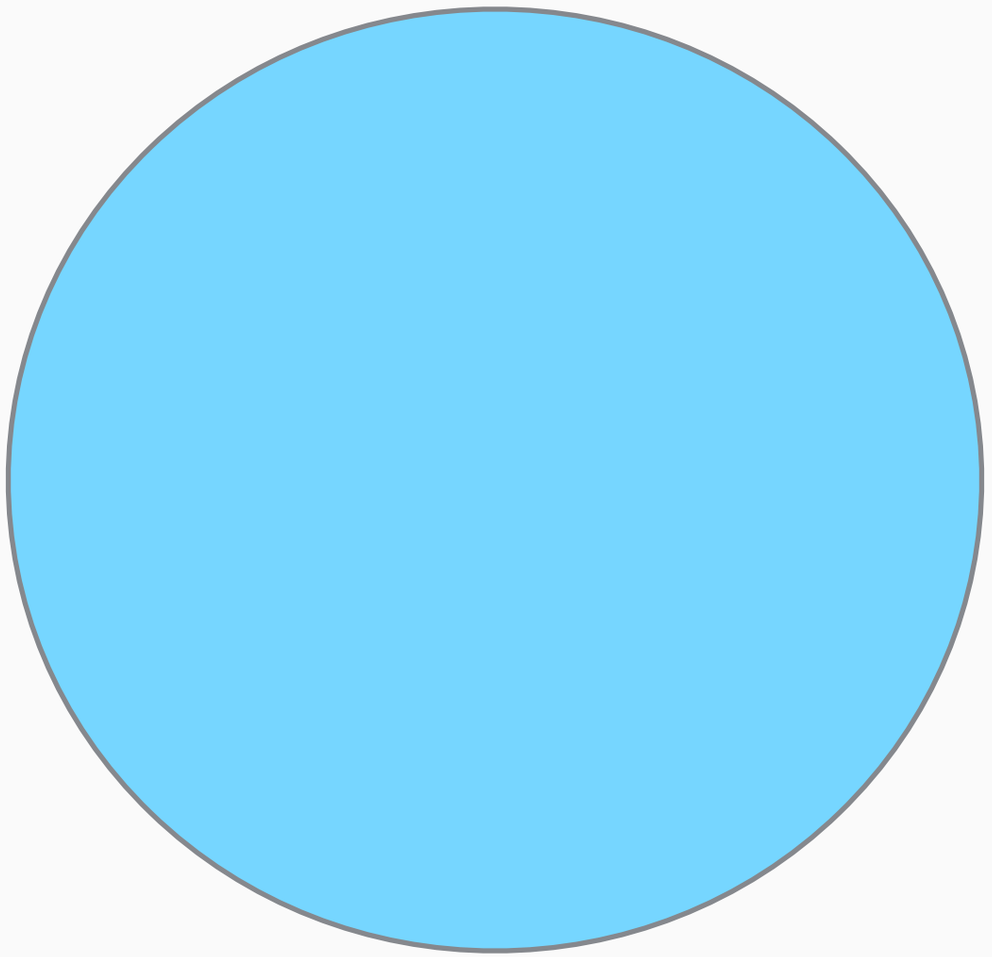

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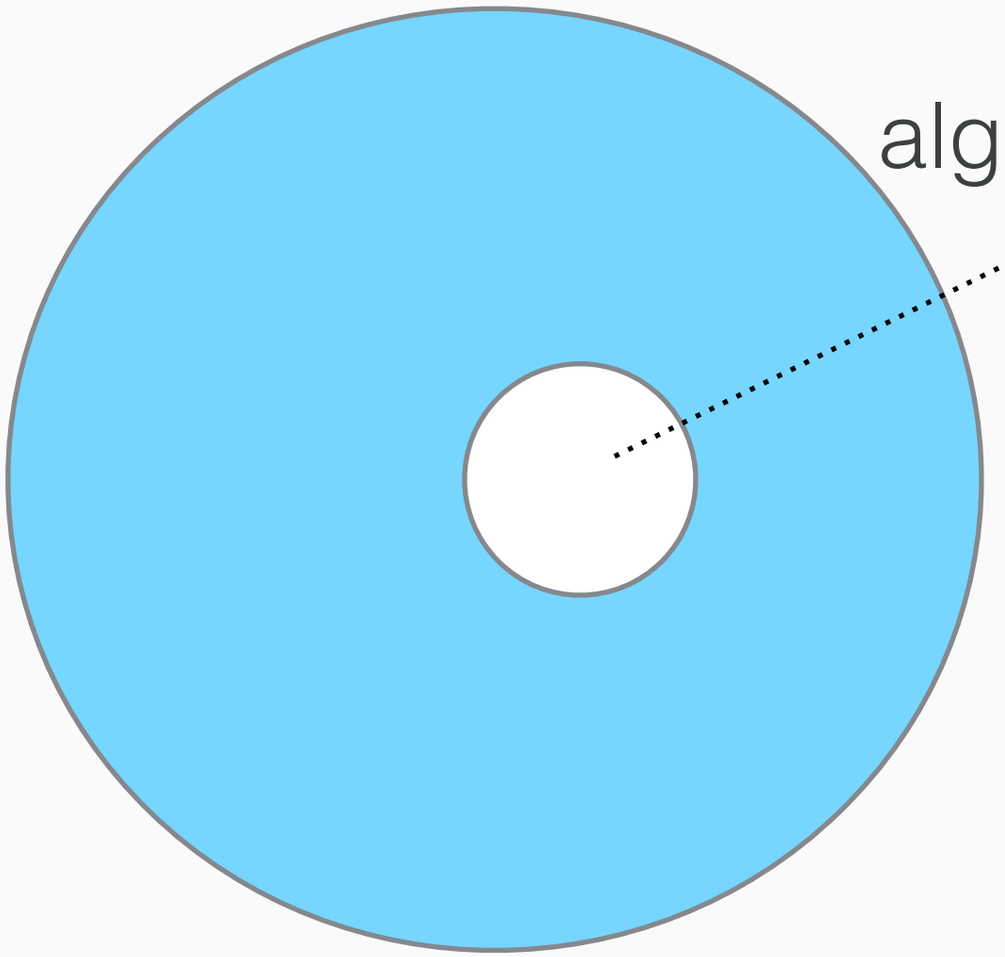

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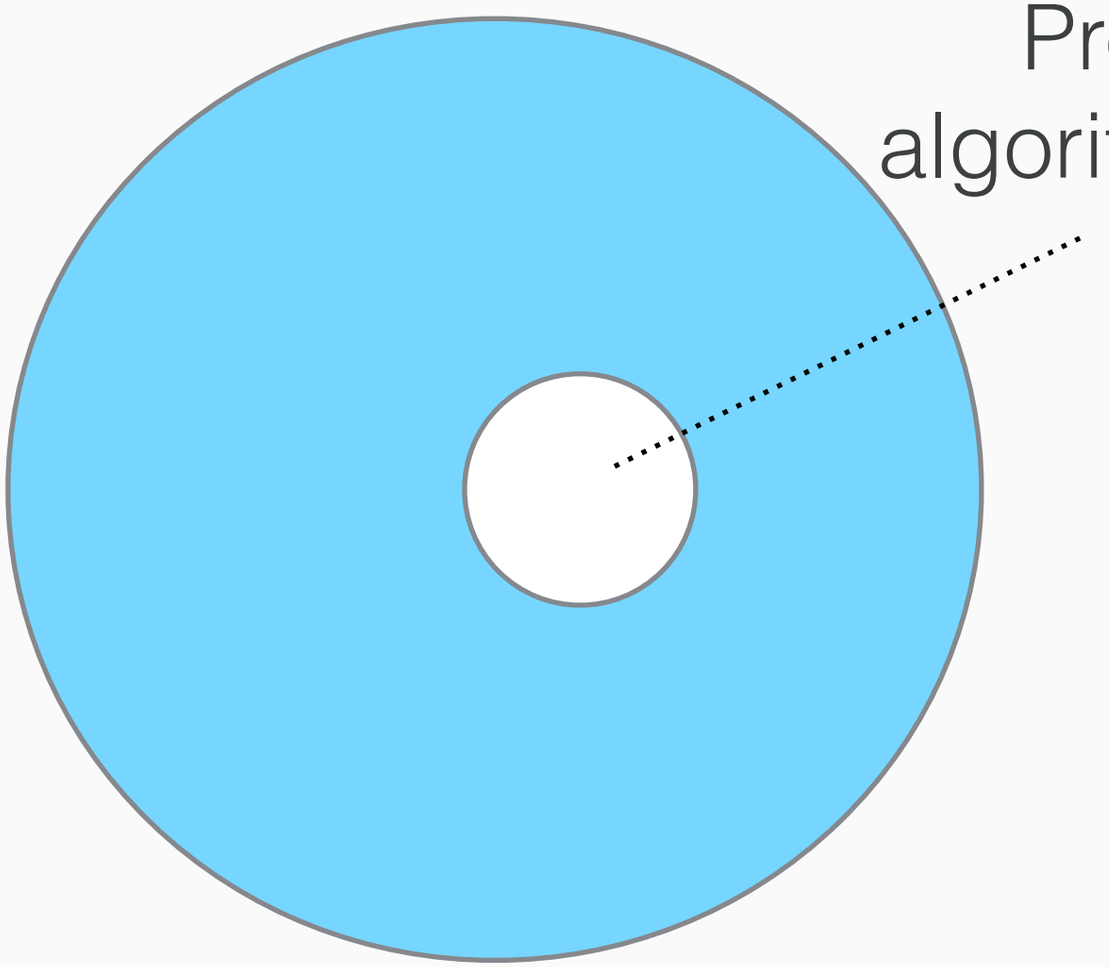

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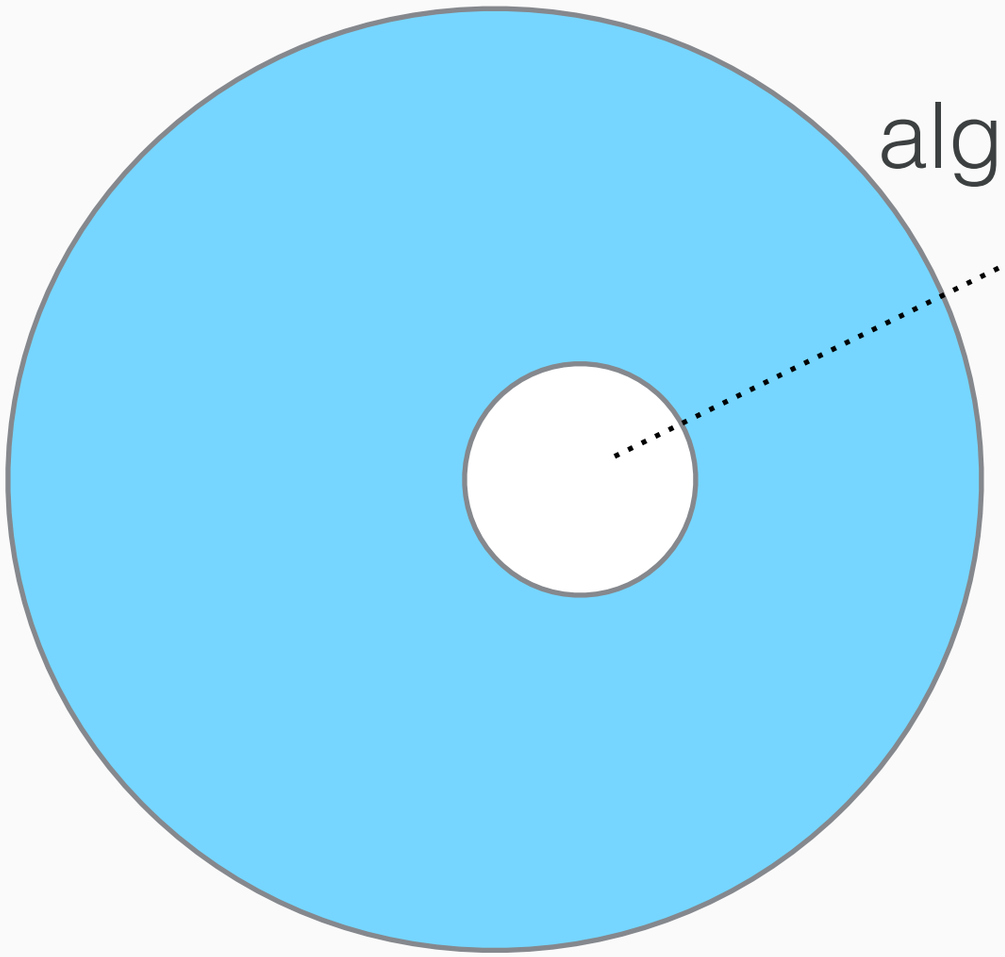

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Q: Is Search in this?

A: Yes! If this list is sorted, we get *log(N)* from Binary Search, if unsorted, *N* from Linear Search

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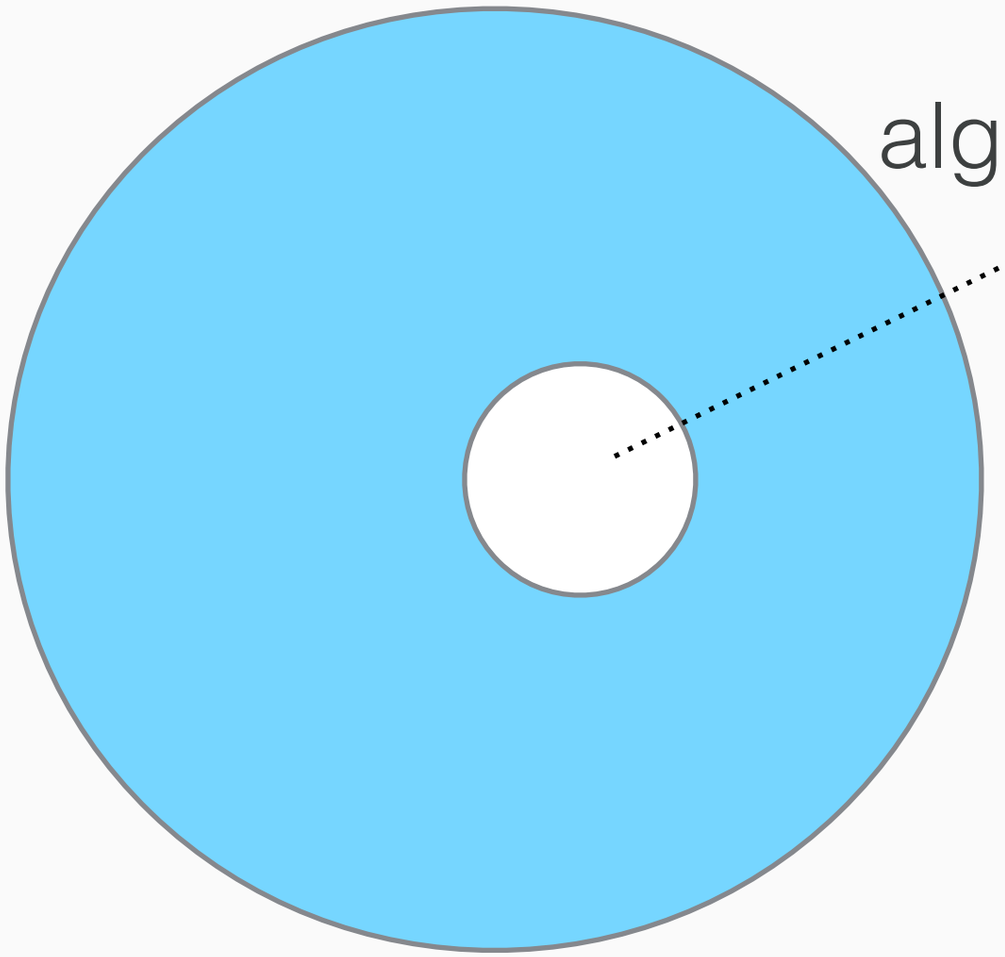

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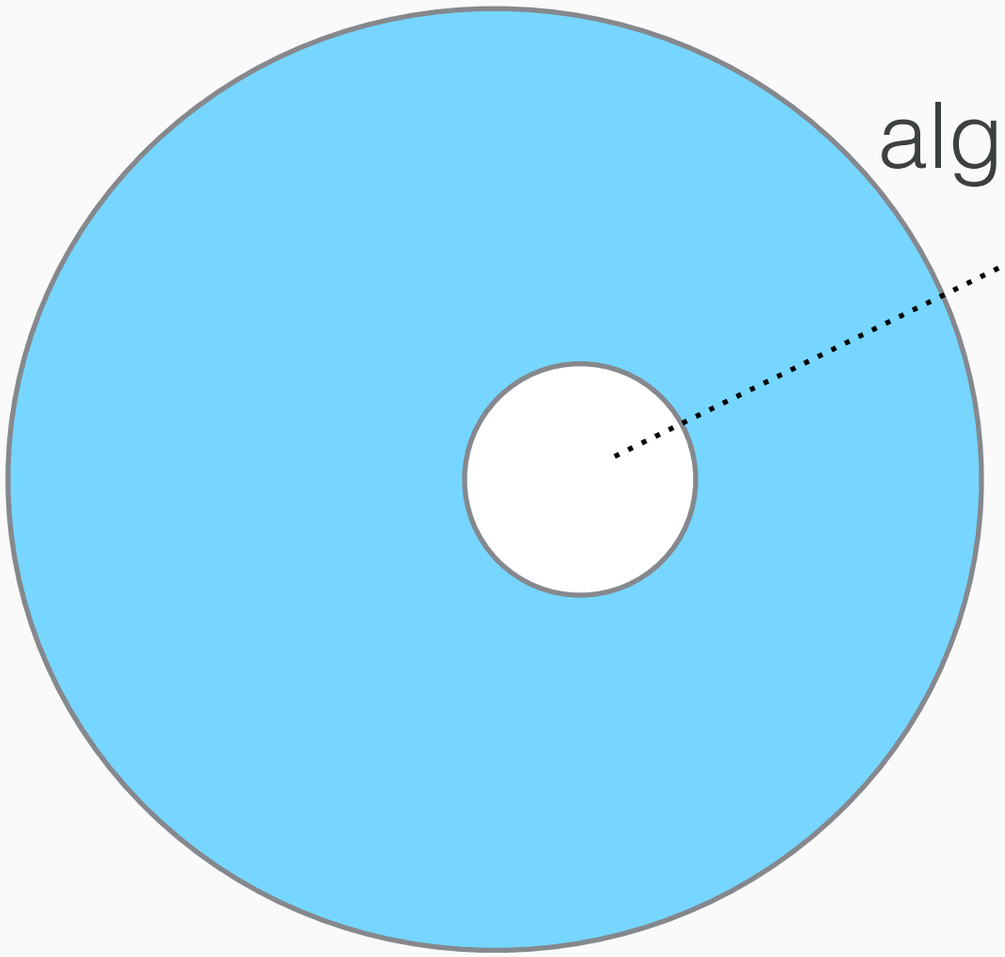

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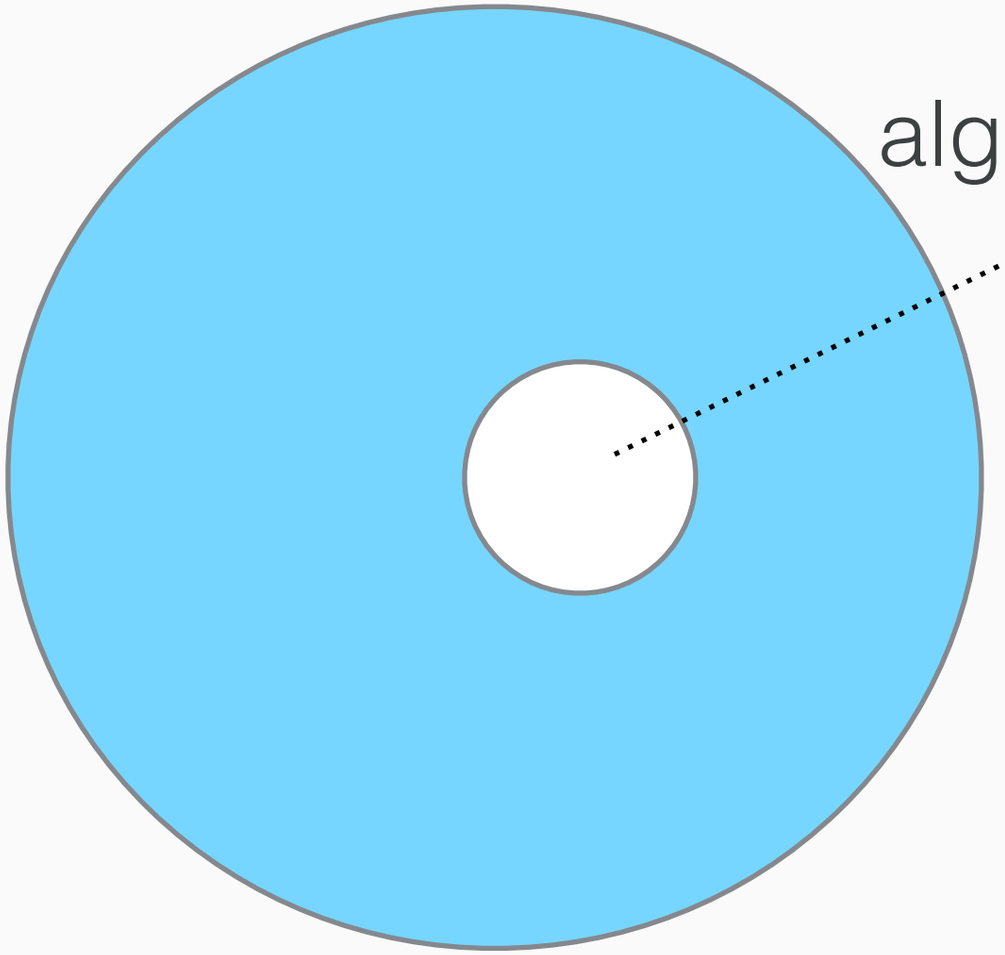

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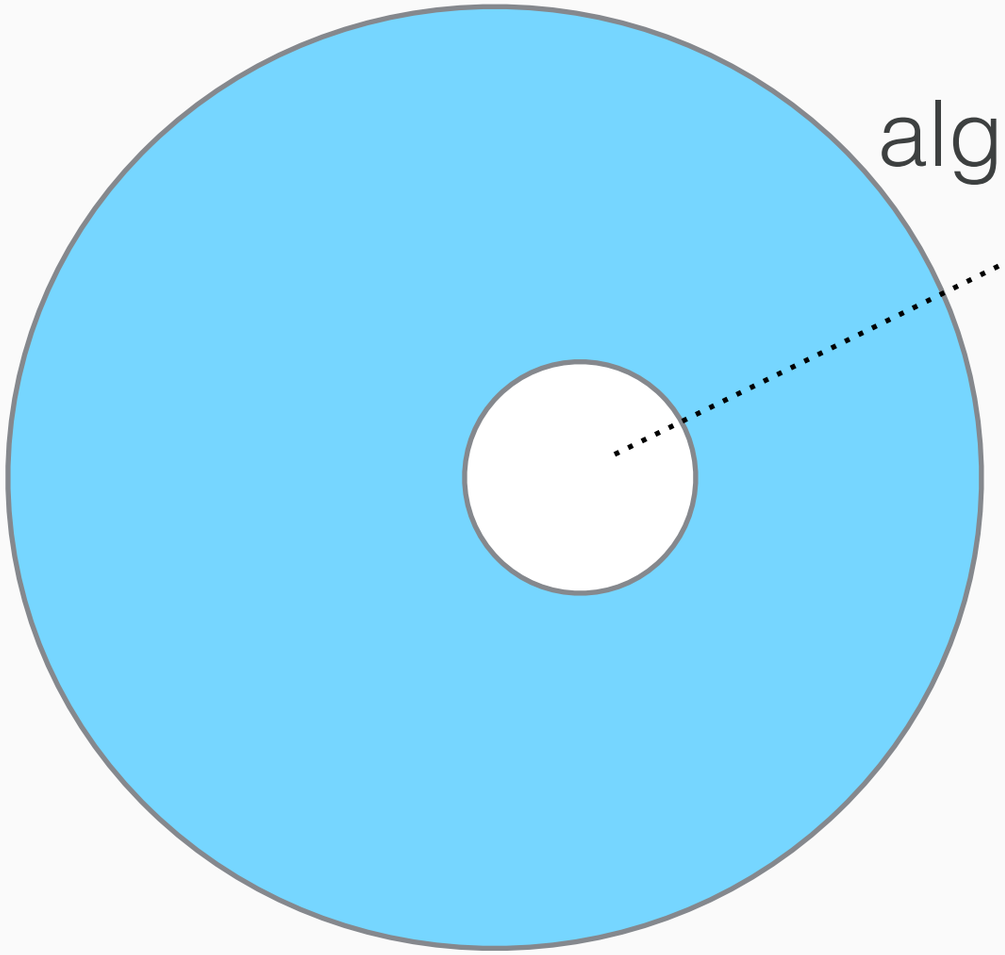

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Example: $5N^3 + 2N^2 + N$

Example: $8N^{27} + 9N^5$

Growth Rates

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We can do this for *all* ***relevant*** *growth rates:*

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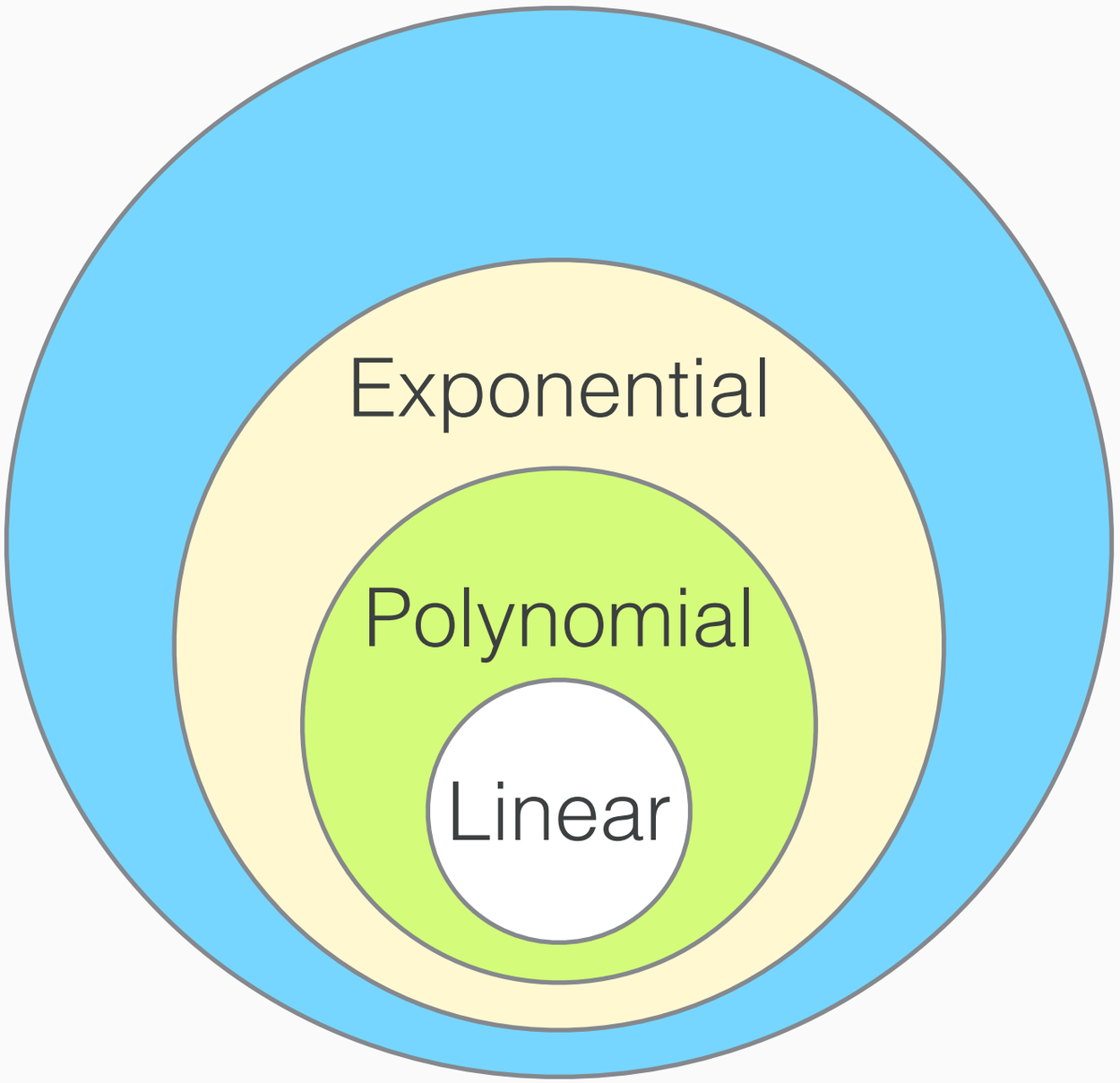
2. Polynomial or faster *(N^5, N^{20}, N, etc.)*
3. Exponential or faster *(2^N, 3^N, etc.)*

Growth Rates



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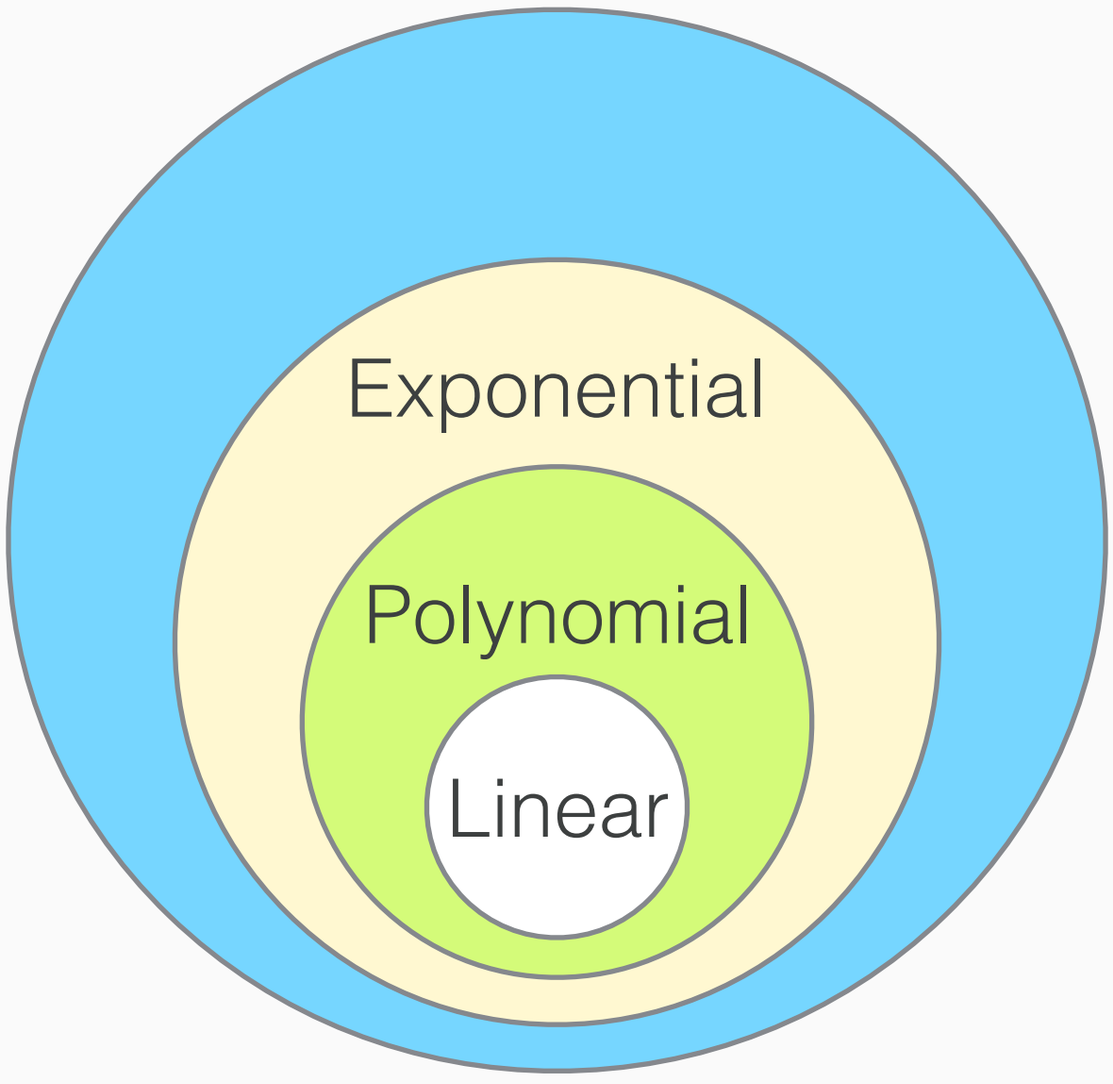
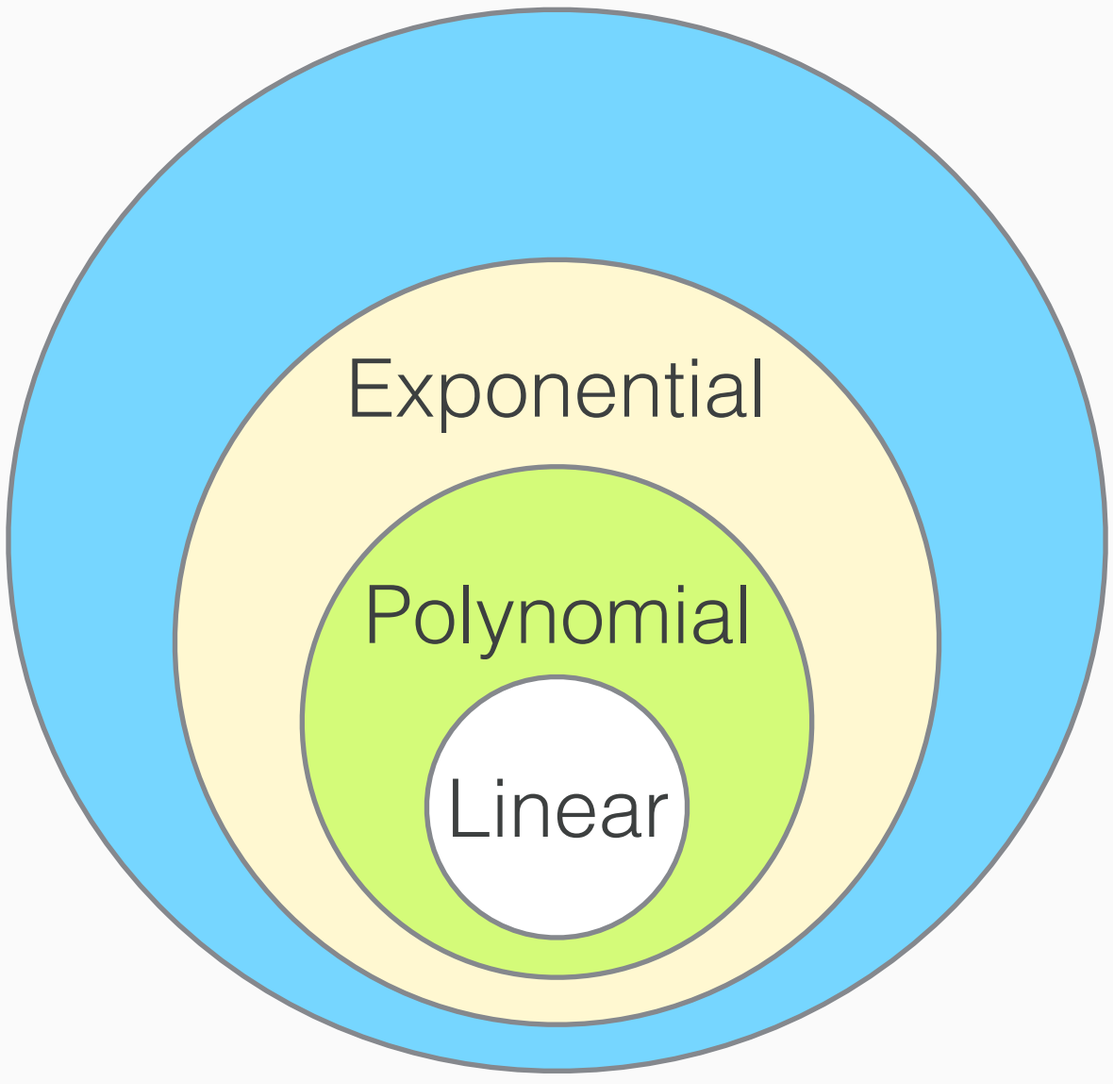


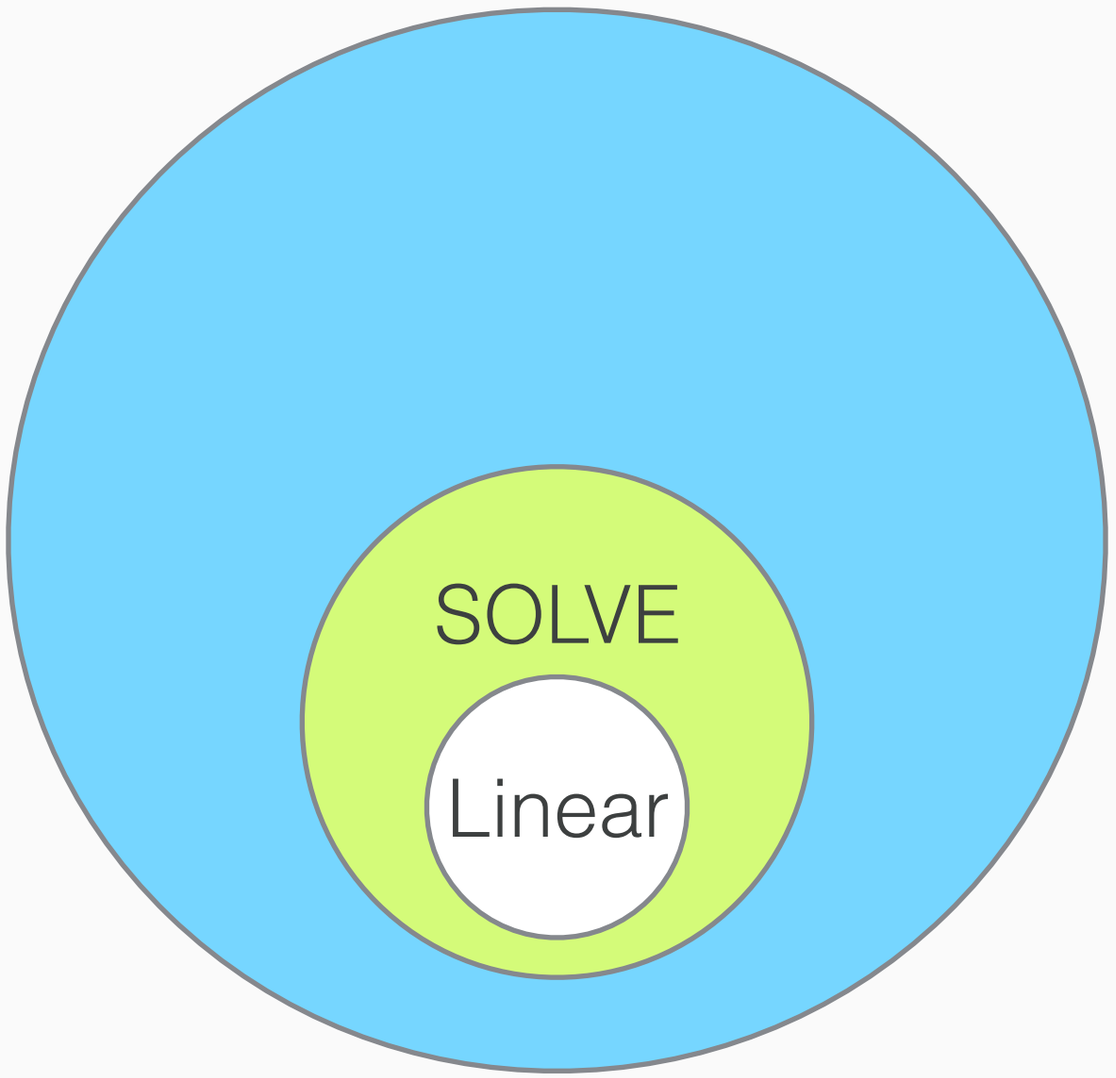
Reminder: these are the problems that have algorithms whose growth rates are *at most* Linear, Polynomial, etc.

Growth Rates



Important: polynomial is green because that's the class of problems we consider **solvable.**

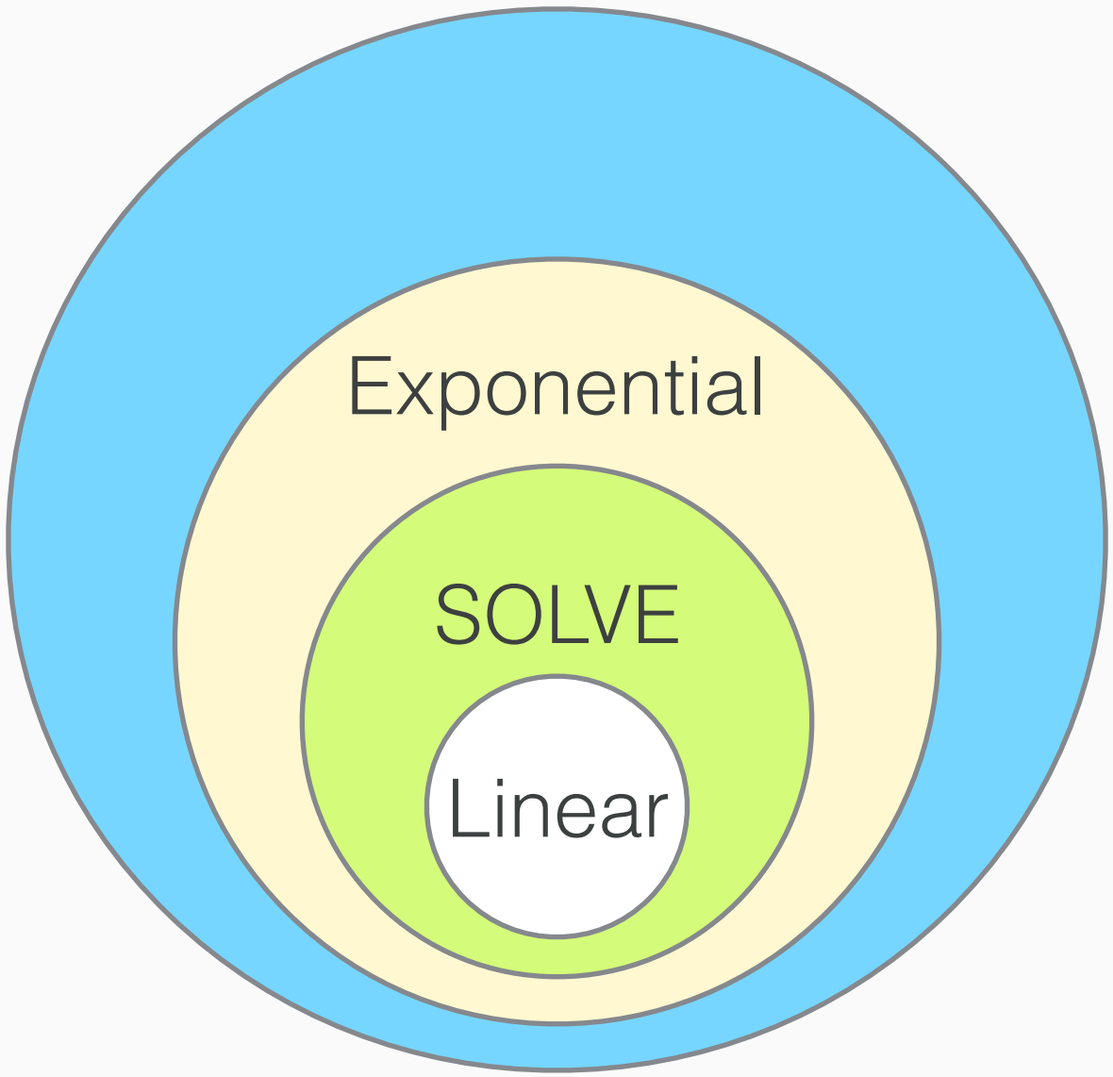
Class: SOLVE



In CS8, we'll call this class, SOLVE

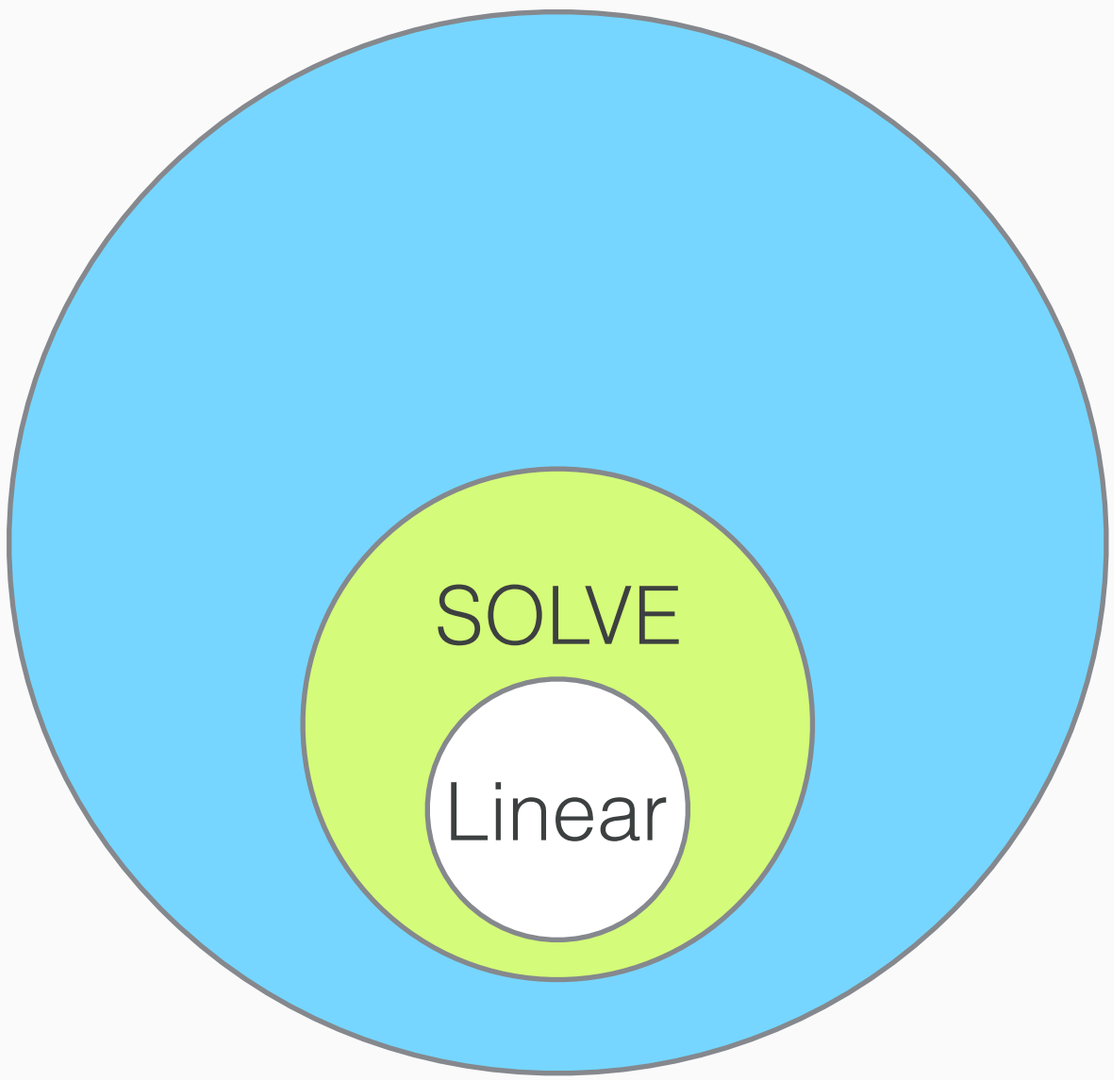
Important: polynomial is green because that's the class of problems we consider **solvable.**

Reminder: Exponentials are BIG



2^{100} = 1,267,650,600,228,229,401,496,703,205,376

Clicker Question:



Q: Is the *Sorting* problem in SOLVE?

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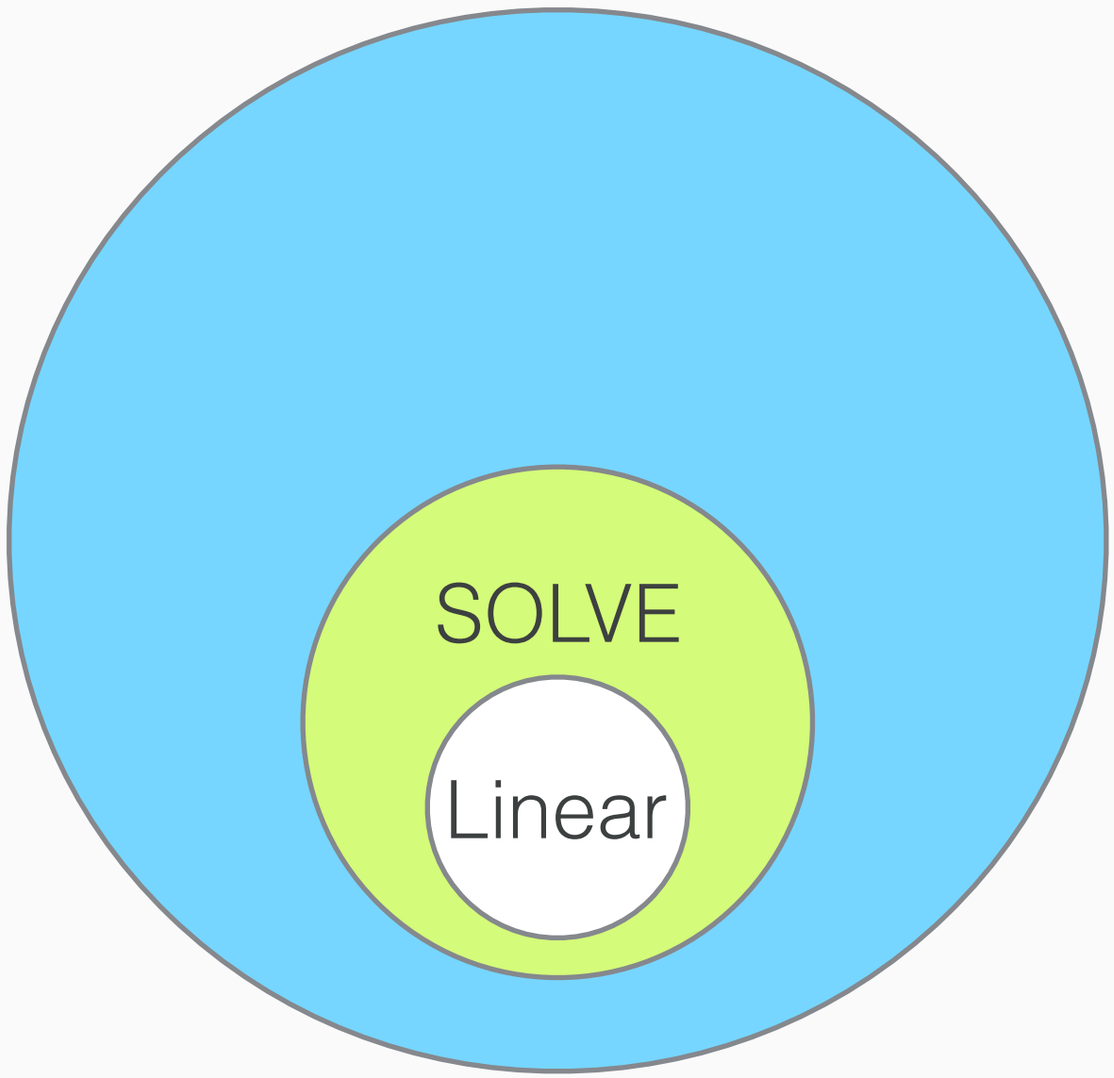


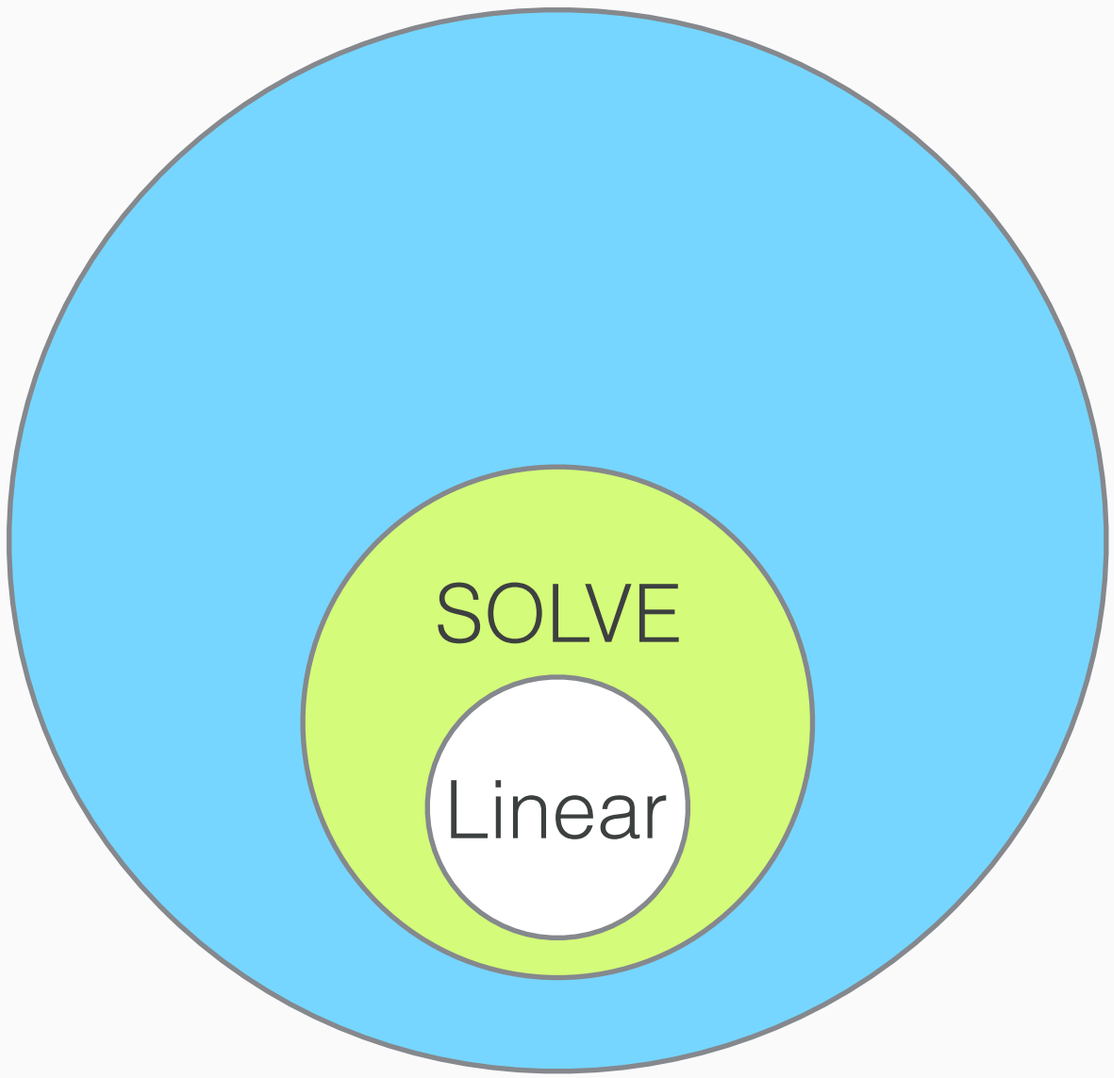
Q: Is the *Sorting* problem in SOLVE?

[A] Yes!

[B] No!

[C] I'm confused :/

Clicker Answer:



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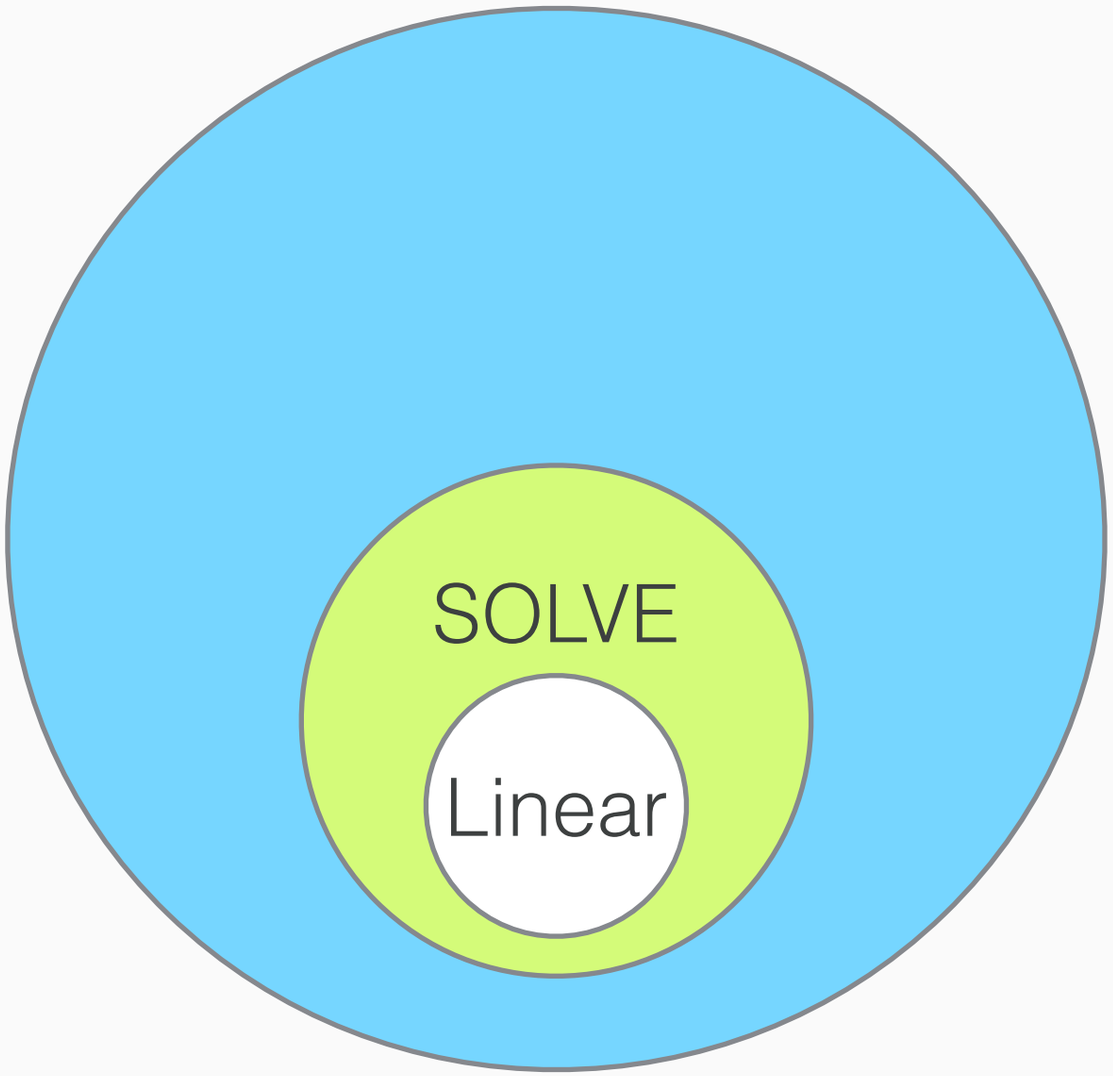

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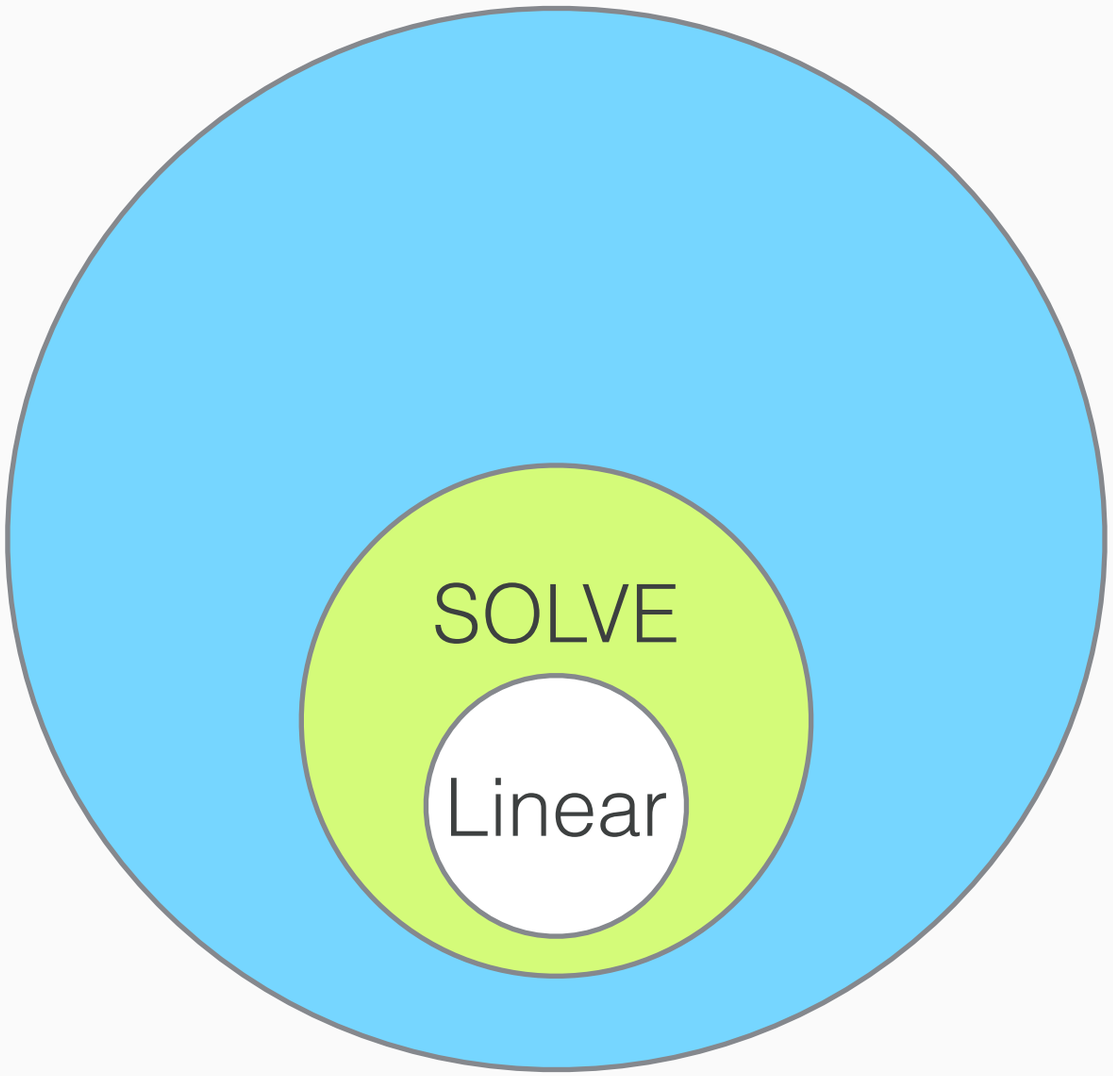
[A] Yes!

[B] No!

Reminder: SOLVE are the problems that have algorithms whose growth rates are *at most* Polynomial.

Clicker Answer:



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Selection Sort: N^2

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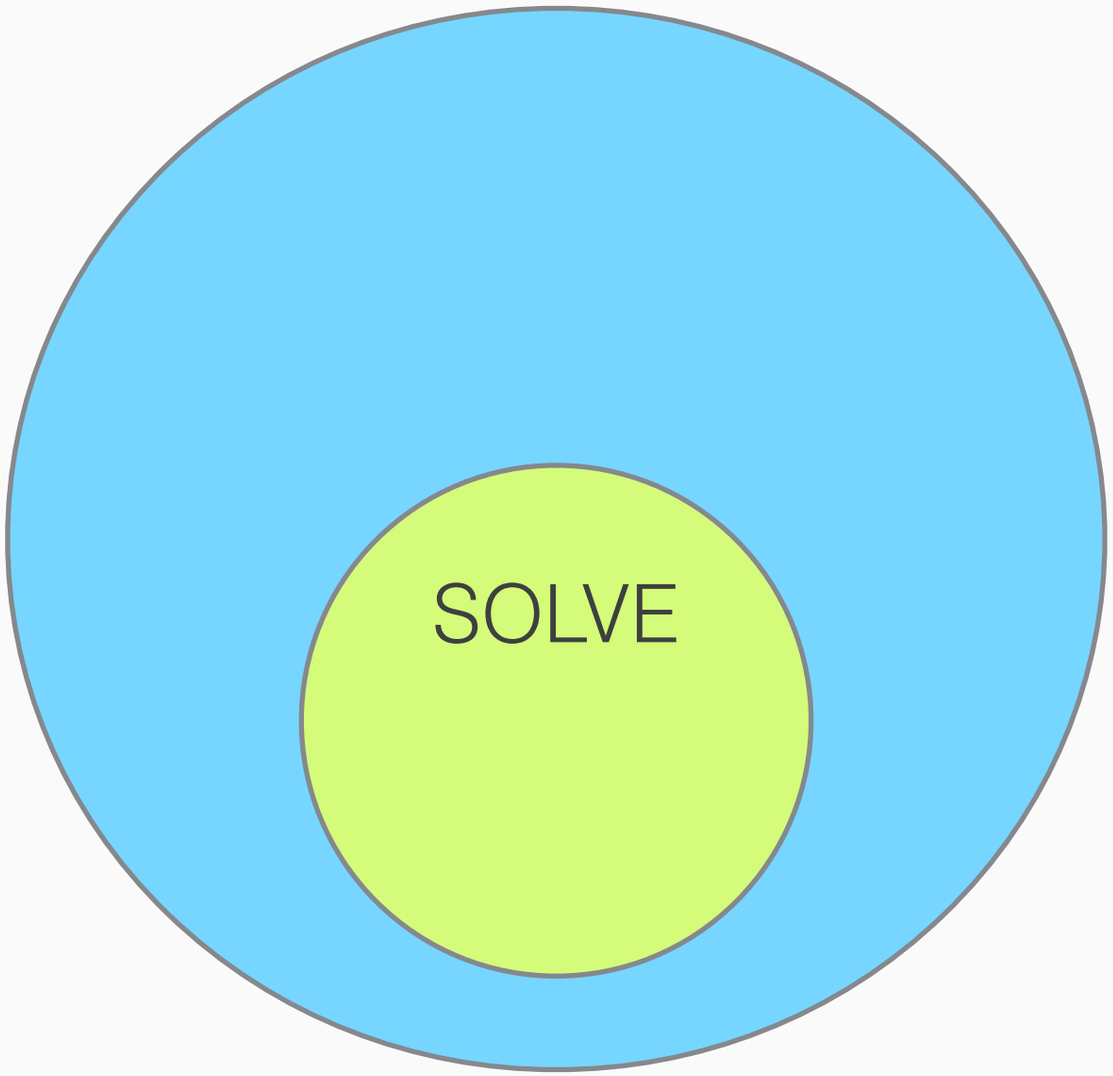
SOLVE



Q: Can we solve a problem efficiently?

A: Is it in SOLVE?

Reminder:

- Problem Specification:
  - INPUT: some things
  - OUTPUT: some true stuff about the things

Reminder:

- Problem Specification:
  - INPUT: some things
  - OUTPUT: some true stuff about the things
- Example:
  - INPUT: A Sudoku board
  - OUTPUT: Solution to the Sudoku board

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Another View: Verification

- Verification Example:
  - INPUT: An empty Sudoku board, a proposed solution to that Sudoku board
  - OUTPUT: True if the Sudoku board is a correct solution

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Another View: Verification

- Another Verification Example:
  - INPUT: An empty Crossword, a proposed solution to that Crossword
  - OUTPUT: True if the filled out Crossword board is a correct solution

Another View: Verification

- Another Verification Example:
  - INPUT: A list, a proposed sorting of that list
  - OUTPUT: True if the proposed sorting is actually in sorted order.

Discuss!

- In light of recent events consider how *making the perfect single move* in the Game Go can be pitched as a verification problem!

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Talk with your neighbors for a minute or two

Discuss!

- In light of recent events consider how *making the perfect single move* in the Game Go can be pitched as a verification problem!

INPUT: A configuration of the Go board, a Go move

OUTPUT: True if the move is the perfect move.

Another Class: VERIFY

The class of problems VERIFY is the set of problems where we can *verify* solutions