Unit 7: Theory

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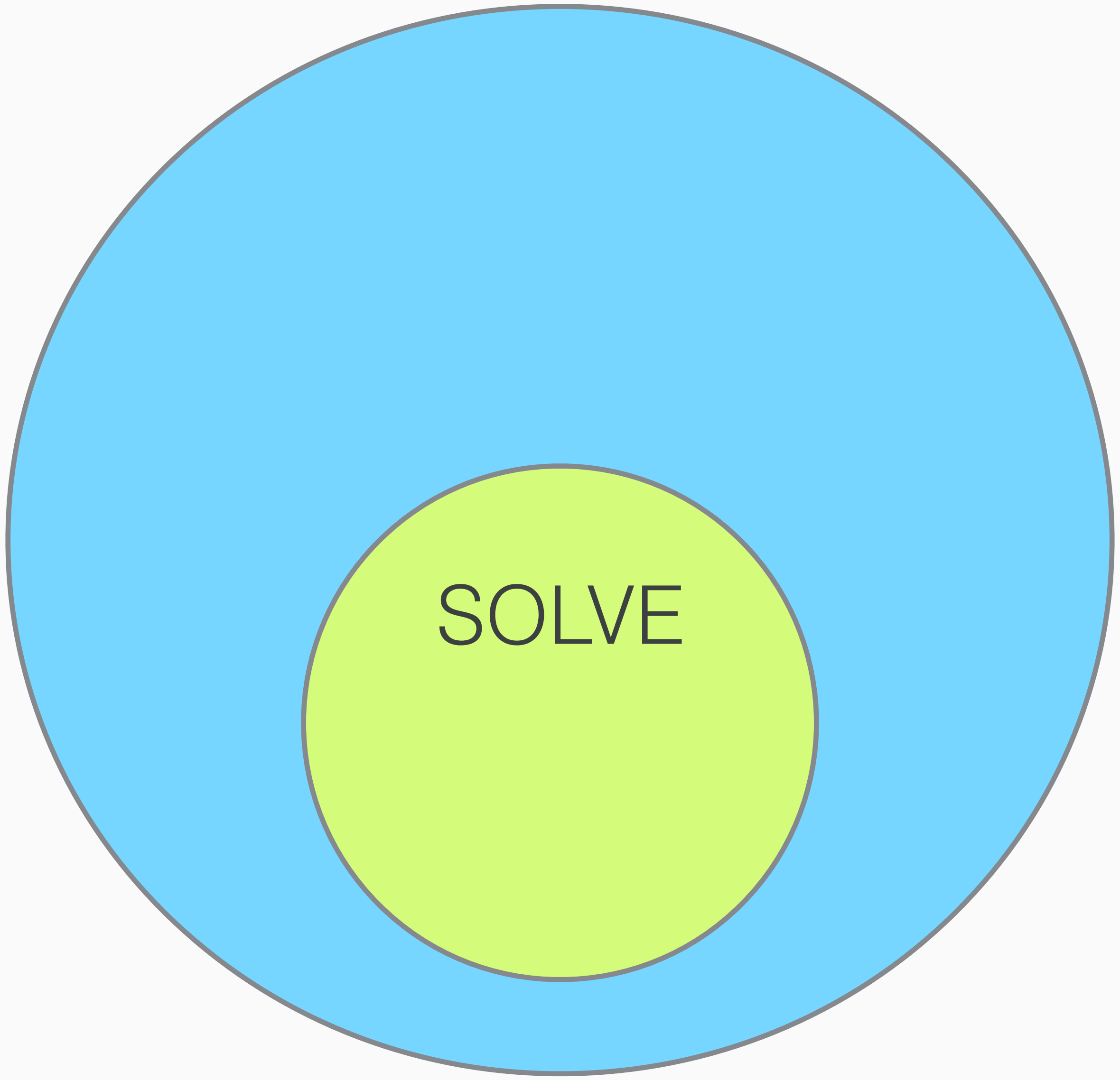
March 18th, 2016

As Promised:

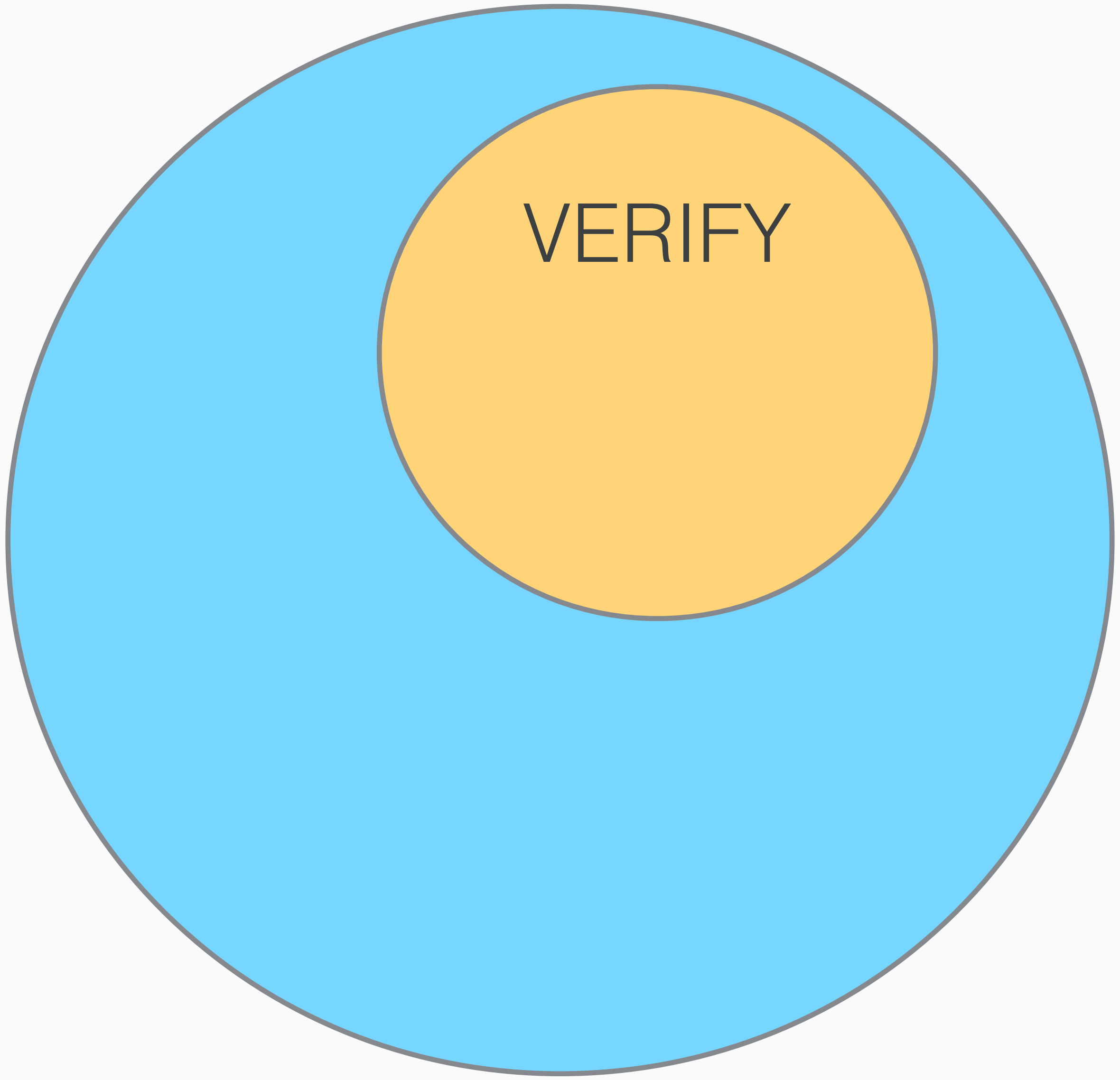
Any Midterm questions?

SOLVE



Problems that we can solve efficiently

VERIFY



Problems for which candidate solutions can be verified efficiently

This Is It!

Q: Given *any* problem known to be in VERIFY, do we know anything about its status relative to SOLVE?

A: Nope! (so far)

This is considered the most important unanswered question in all of computer science.

Rephrased

Q: If candidate solutions to a problem can be verified efficiently, can the problem also be solved efficiently?

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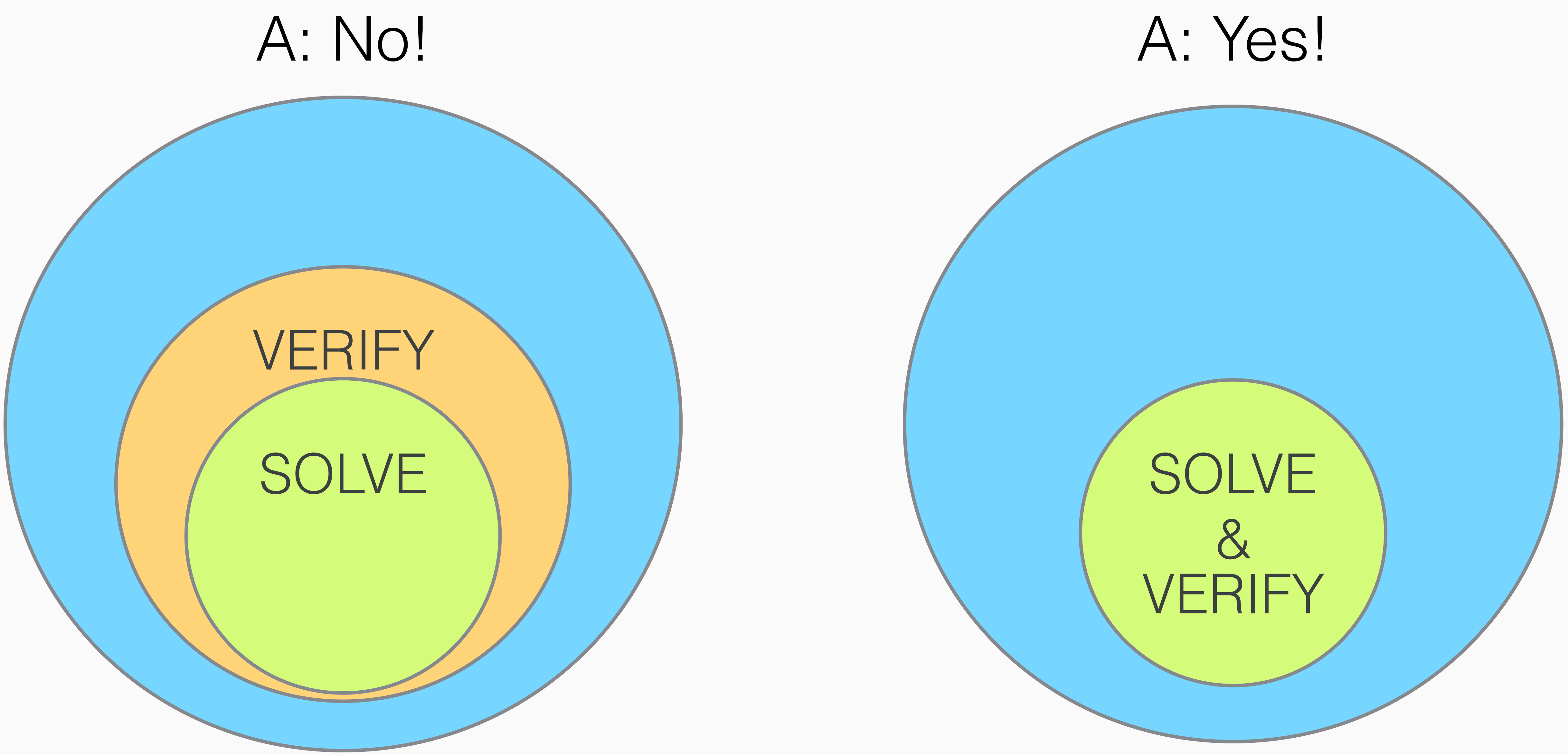
SOLVE versus VERIFY

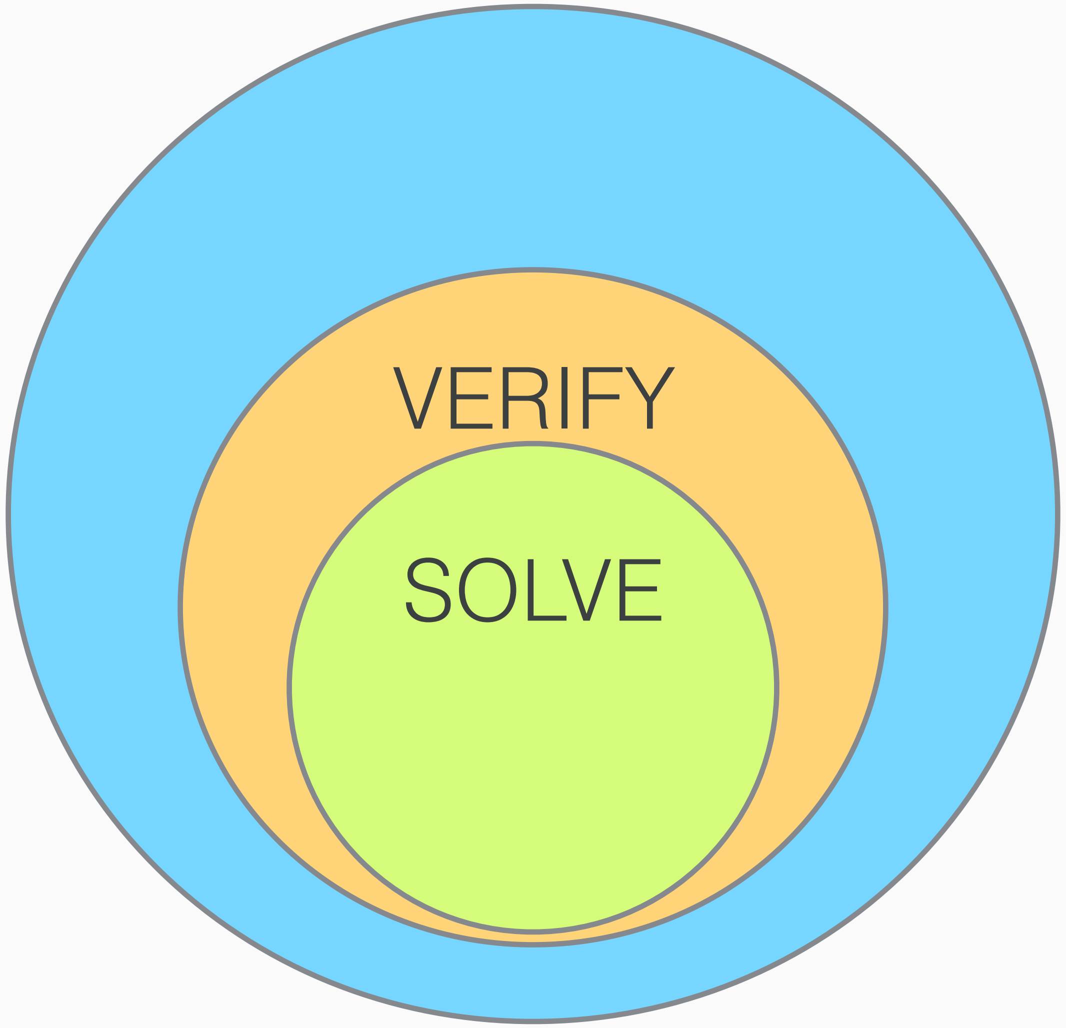


Q: If candidate solutions to a problem can be verified efficiently, can the problem also be solved efficiently?

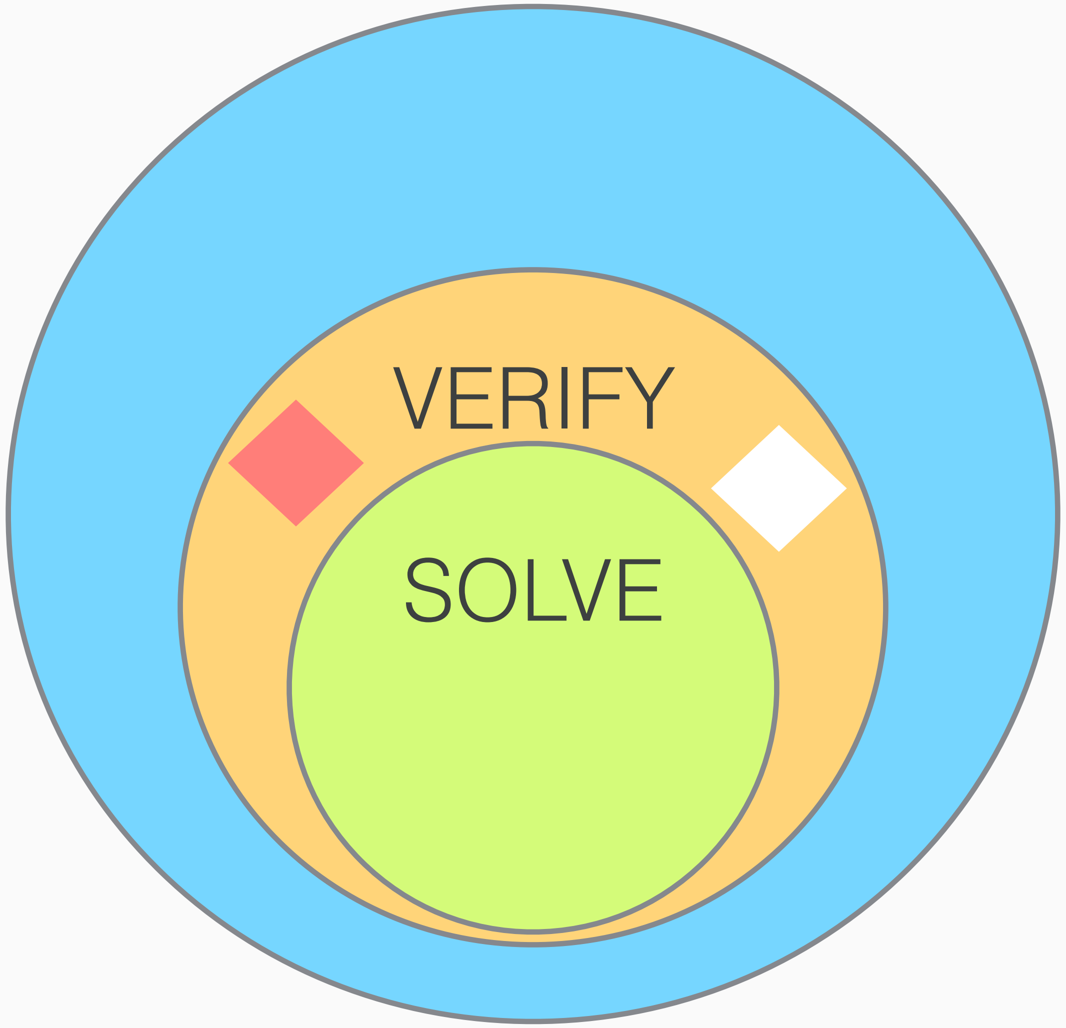
‘The P vs. NP problem has been called “one of the most important problems in contemporary mathematics and theoretical computer science” [60]. That is an understatement. Not only is P vs. NP the defining question of our field; it’s one of the deepest questions ever asked for which we’d know how to recognize an answer.’

-Prof. Scott Aaronson

Some Verification Examples



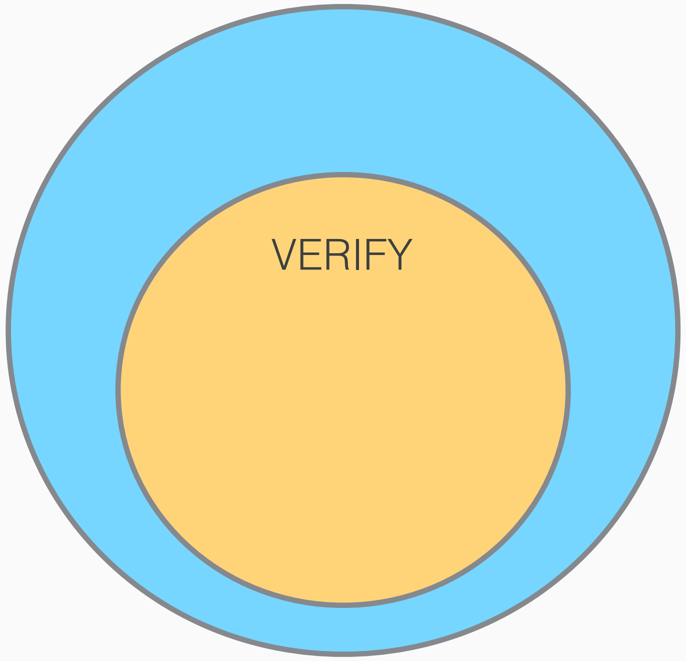
Some Verification Examples



(That we think might not be in SOLVE)

Some Verification Examples

- *Can this logical formula be true?*
  - (Also called Satisfiability)
- Subset Sum
- Circle + Line problems!



Can this logical formula be true?

- *Input:*
  - a logical formula
- *Output:*
  - True if there is some way we can make the formula true
  - False if there is no way we can make the formula true

Can this logical formula be true?

I give you: *AND(P,Q)*

Q: Can this be true?

(our tools are how we set the truth value of *P, Q*)

Can this logical formula be true?

I give you: *AND(P,Q)*

Q: Can this be true?

A: Sure! *P = True, Q = True*

(our tools are how we set the truth value of *P, Q*)

Verification: Can this logical formula be true?

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- *Input:*
  - a logical formula, an assignment to each variable
- *Output:*
  - True if the resulting sentence is True
  - False if the resulting sentence is False

Verification: Can this logical formula be true?

I give you: *AND(P,Q), P = True, Q = False*

Q: Is this True?

Verification: Can this logical formula be true?

I give you: *AND(P,Q), P = True, Q = False*

Q: Is this True?

No!

Can this logical formula be true?

Build The Truth Table! (Brute Force)

1. *Build the truth table for the logical formula*
2. *Check to see if it has any row that is True.*
3. *If it does, report True.*
4. *If not, report False.*

Build the Table!

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1. Build the Truth Table
2. Check to see if any row is True.
3. If so, report True.
4. Otherwise, report False.

Q: Growth Rate?

(*N* is number of variables..)

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1. Build the Truth Table
2. Check to see if any row is True.
3. If so, report True.
4. Otherwise, report False.

Q: Growth Rate?

(*N* is number of variables..)

A: 2^N

P	*Q*	*R*	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Verification: Can this logical formula be true?

- *Input:*
  - a logical formula, an assignment to each variable
- *Output:*
  - True if the resulting sentence is True
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Algorithm: plug in the assignments, check if True!

Verification: Can this logical formula be true?

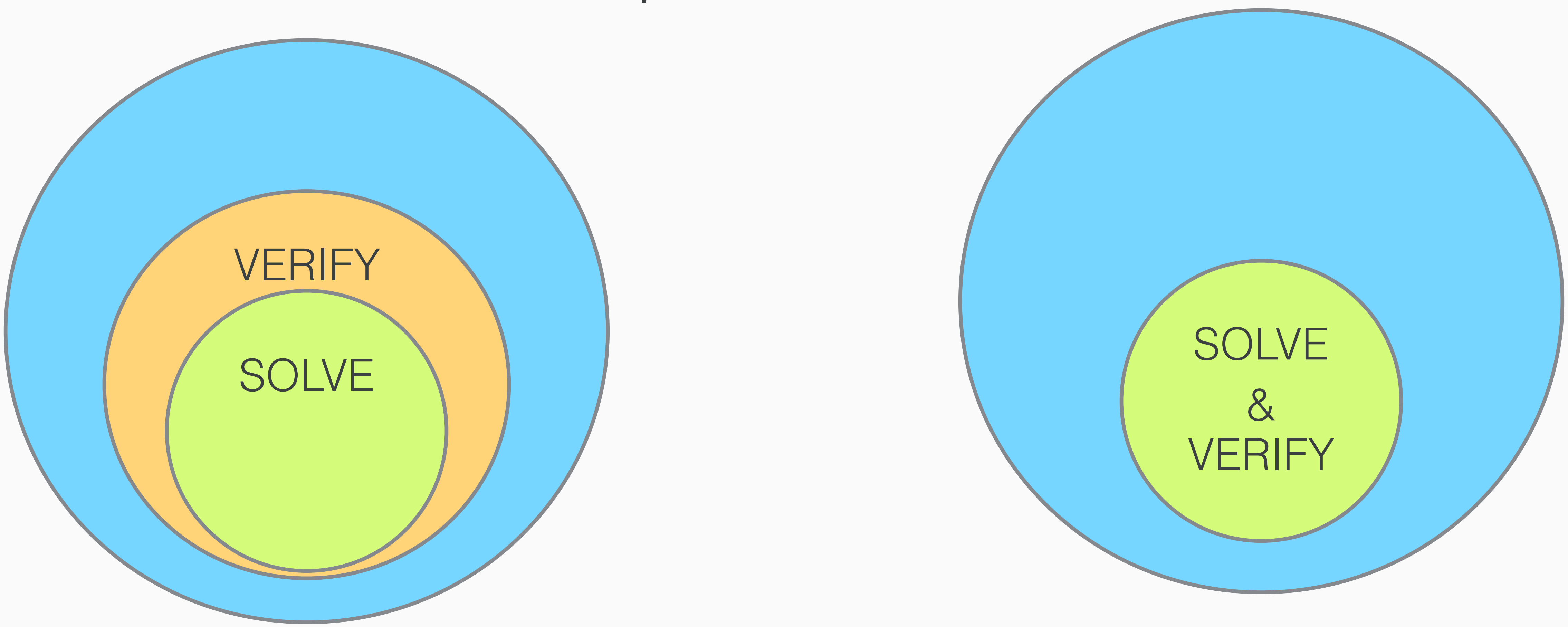
- *Input:*
  - a logical formula, an assignment to each variable
- *Output:*
  - True if the resulting sentence is True
  - False if the resulting sentence is False

Growth Rate: N

Algorithm: plug in the assignments, check if True!

Satisfiability (SAT)

- *Can **verify** with N operations*
- *Can **solve** with 2^N operations.*



A: No

A: Yes

Problem: Subset Sum

- *Input:*
  - Some numbers! Like: 5,2,14,6,12,1,3
- *Output:*
  - True if some of the given numbers add to 10
  - False if none of the numbers add to 10

Clicker Question!

- *Input:*
  - Some numbers! Like: 5,2,14,6,12,1,3
- *Output:*
  - True if some of the given numbers add to 10
  - False if none of the numbers add to 10

[A] TRUE

[B] FALSE

Q: What do we report for 5,2,14,6,12,1,3?

Clicker Answer!

- *Input:*
  - Some numbers! Like: 5,2,14,6,12,1,3
- *Output:*
  - True if some of the given numbers add to 10
  - False if none of the numbers add to 10

[A] TRUE

[B] FALSE

Q: What do we report for 5,2,14,6,12,1,3?

Verification: Subset Sum

- *Input:*
  - Some numbers! Like: 5,2,14,6,12,1,3
  - A proposed solution (e.g. 5,1,3)
- *Output:*
  - True if the proposed solution adds to 10
  - False if the proposed solution does not add to 10

Verification: Subset Sum

- *Input:*
  - Some numbers: 5,2,14,6,12,1,3
  - Proposed solution: 2,6,4
- *Output:*
  - True if 2,6,4 adds to 10
  - False if 2,6,4 adds to10

Verification: Subset Sum

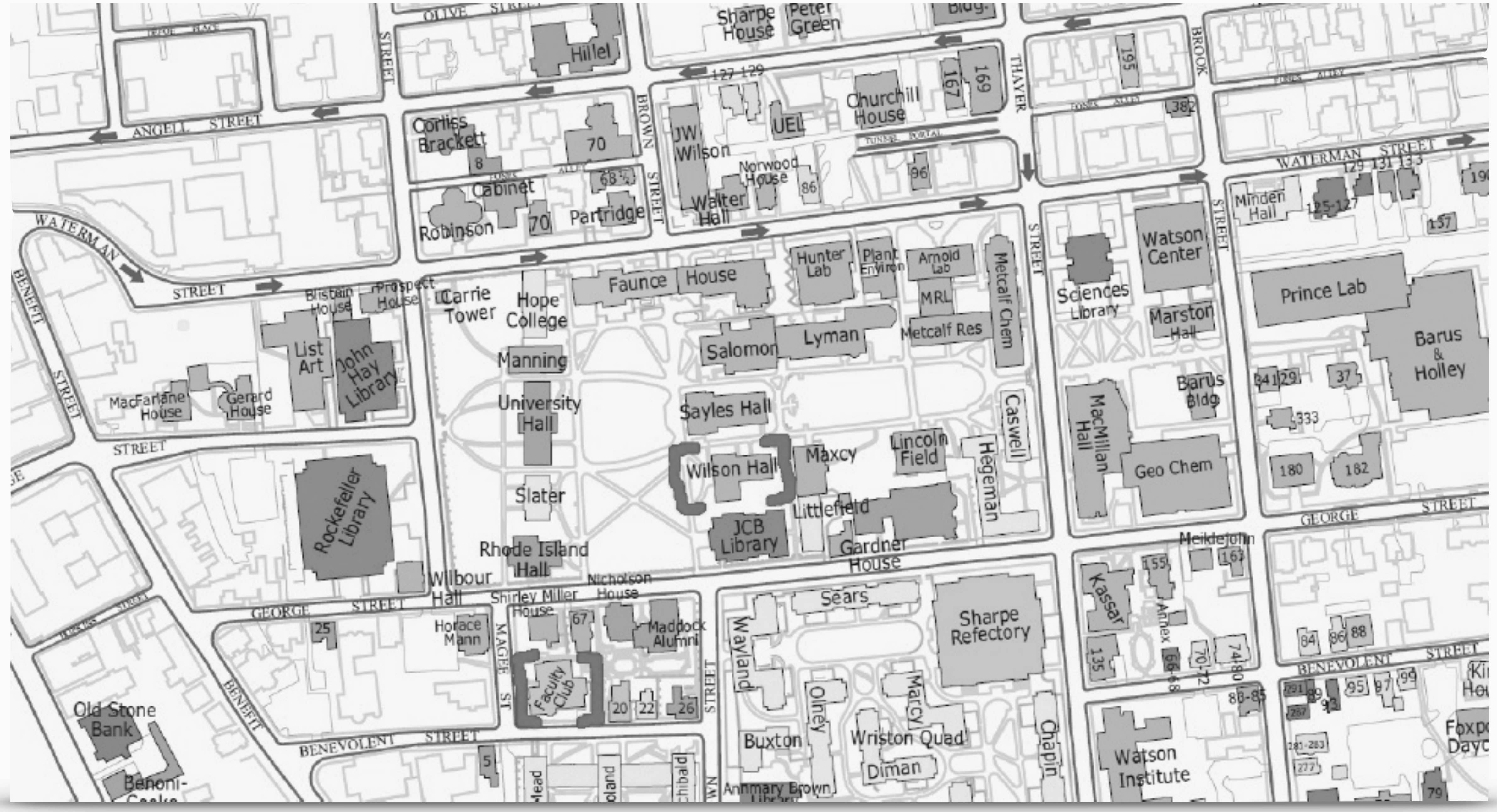
- *Input:*
  - Some numbers: 5,2,14,6,12,1,3
  - Proposed solution: 2,6,4
- *Output:*
  - True if 2,6,4 adds to 10
  - **False** if 2,6,4 adds to10

Verification: Subset Sum

- *Input:*
  - Some numbers: 5,2,14,6,12,1,3
  - Proposed solution: 2,6,4
- *Output:*
  - True if 2,6,4 adds to 10
  - **False** if 2,6,4 adds to10

But: no known efficient solution for solving this problem!

Circles + Lines!



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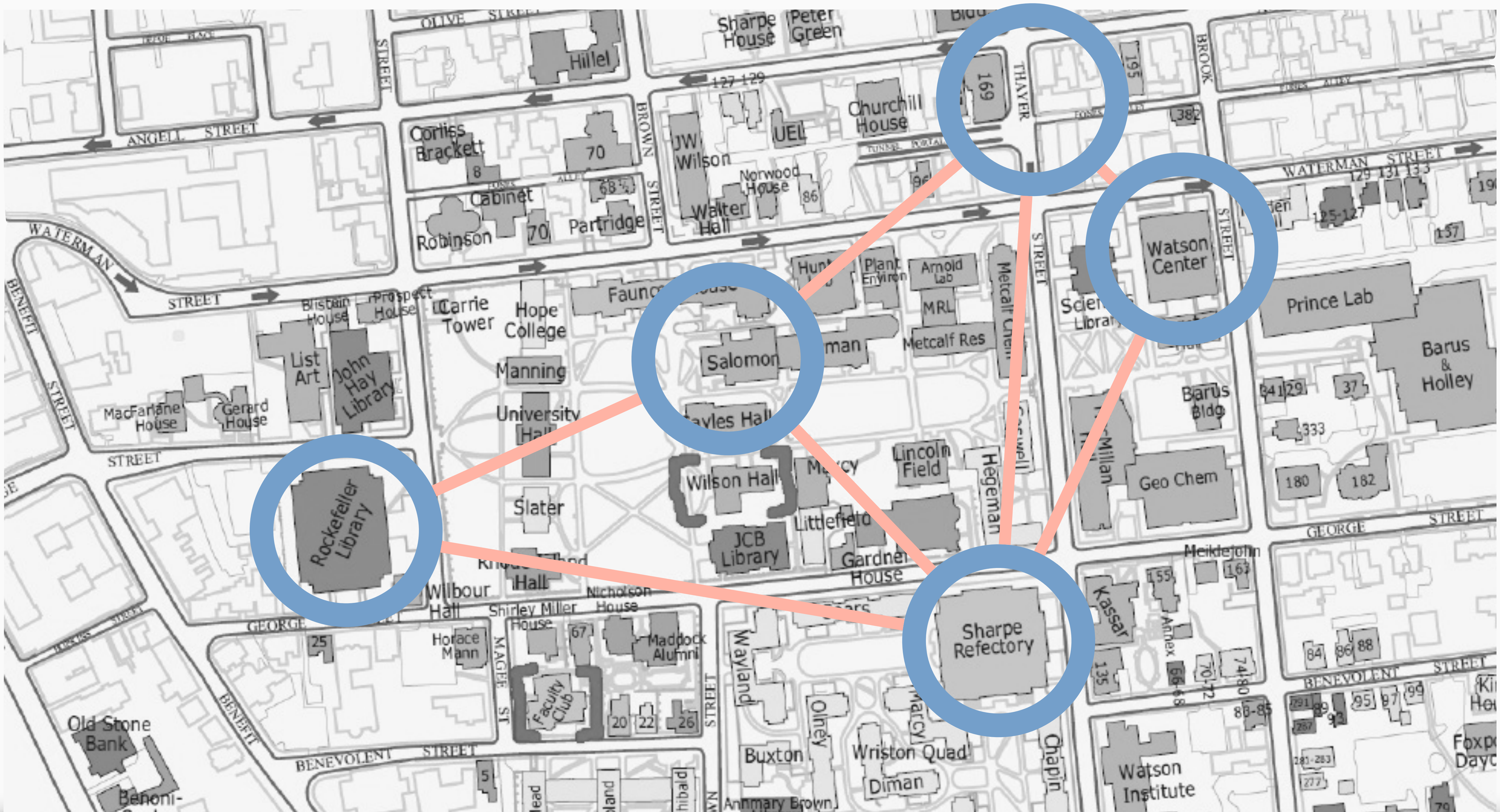

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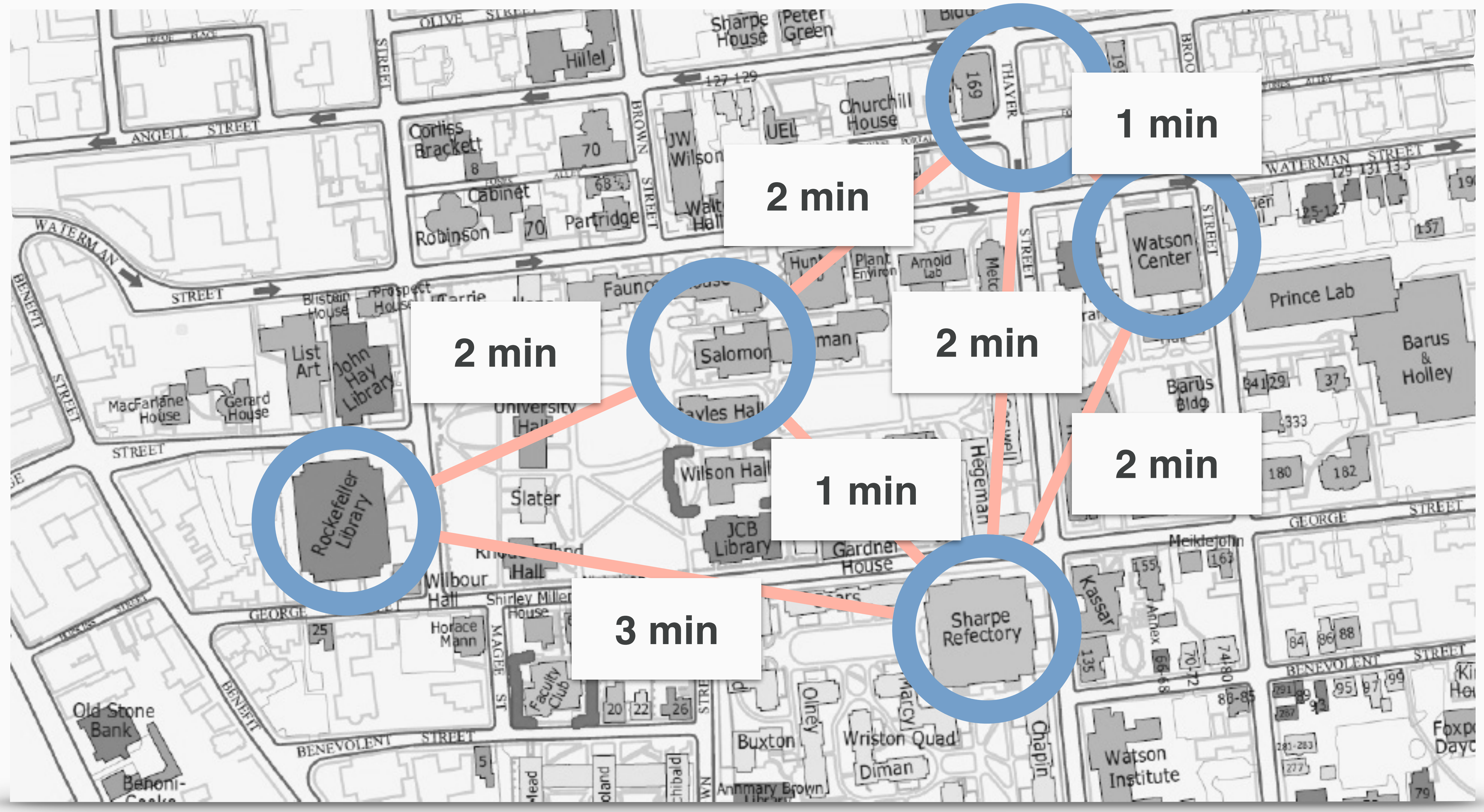

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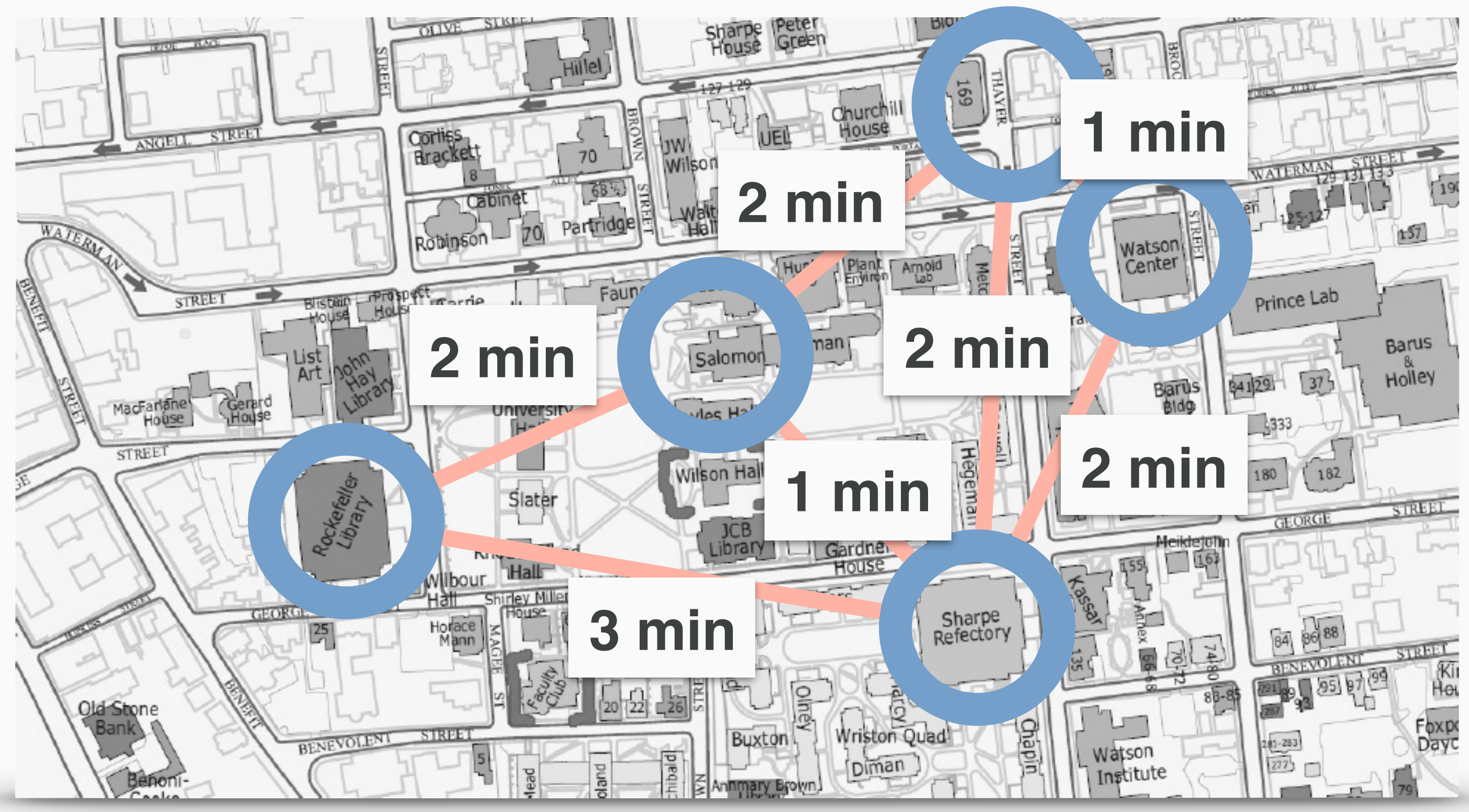
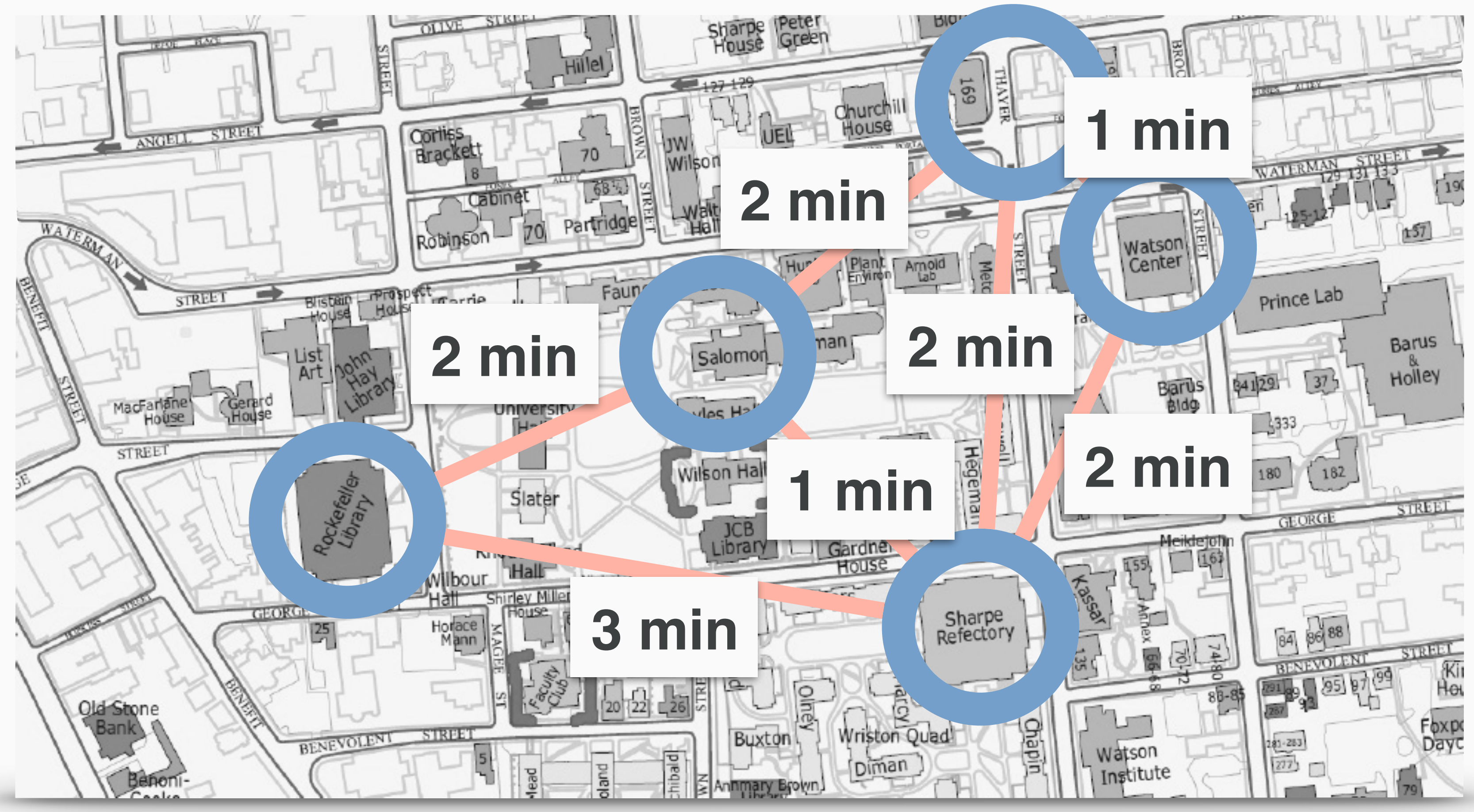


Circle: an object (a location in this case)

Line: a relation between objects (tram, in this case)

“Graph”



Circle: an object (a location in this case)

Line: a relation between objects (tram, in this case)

Reflection

- Q: If candidate solutions to a problem can be verified efficiently, can the problem also be solved efficiently?
- Verification Examples:
  - Can this logical formula be true?
  - Subset Sum
  - Circle + Line problems!

A Disclaimer

Takeaway

There are problems that are unsolvable, period.

Overview: The Uncomputable

- Some problems *have no correct algorithm*.
- Closely related to:
  - Self-reference
  - Paradoxes
  - Infinity

Things That Describe Themselves

“I am a sentence consisting of eight words”

Things That Describe Themselves

Another view:

“X” Is a sentence with eight words in it.

Things That Describe Themselves

Another view:

“X” Is a sentence with eight words in it.

X = “The old man the boat”

“The old man the boat” Is a sentence with eight words in it.

Things That Describe Themselves

Another view:

“X” Is a sentence with eight words in it.

X = “The old man the boat”

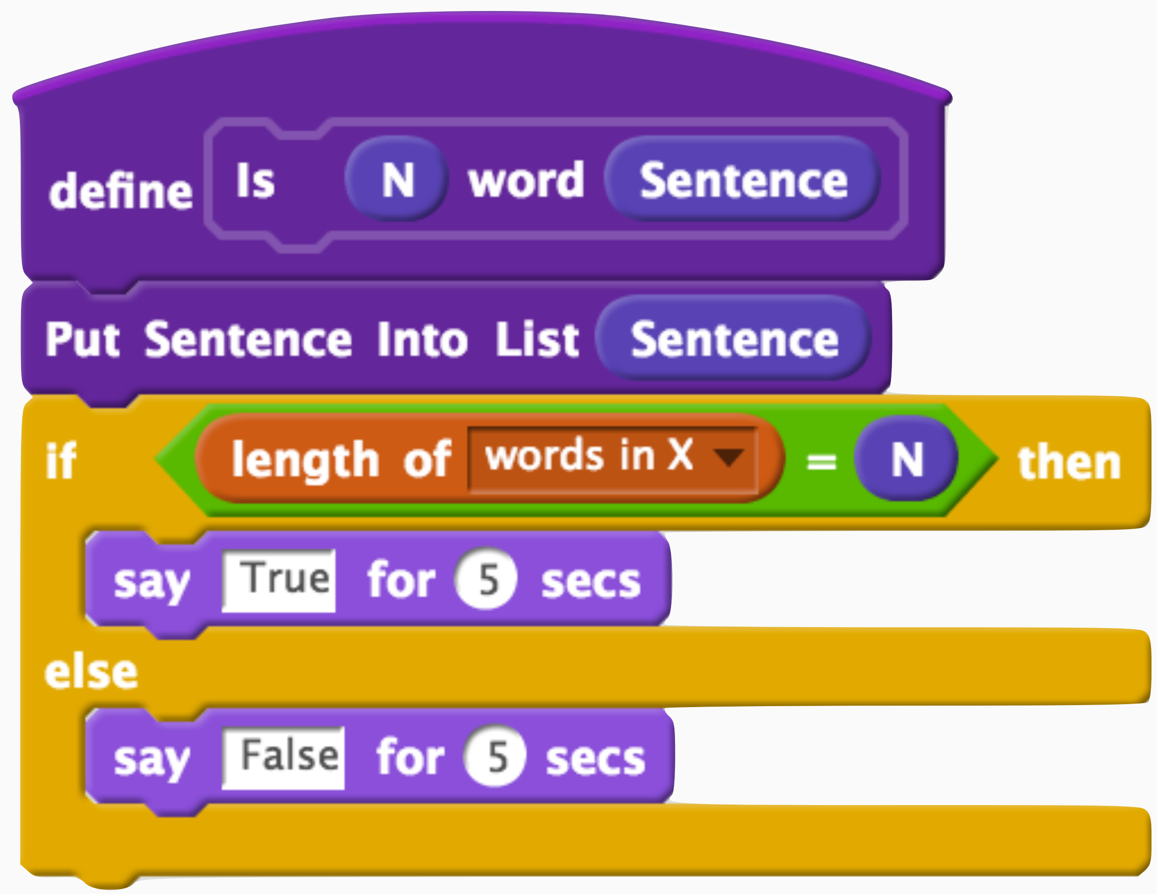
“The old man the boat” Is a sentence with eight words in it.

False

Things That Describe Themselves

Another view:

Does "sentence" have N words?



Things That Describe Themselves

Q: What about things that lie about themselves?

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“This sentence is False”

Things That Describe Themselves

Q: What about things that lie about themselves?

"This sentence is False"

True

Things That Describe Themselves

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Contradiction

Things That Describe Themselves

Q: What about things that lie about themselves?

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Things That Describe Themselves

Q: What about things that lie about themselves?

"This sentence is False"

(so the content of the sentence has to not be true)

True

False

Contradiction

True

Things That Describe Themselves

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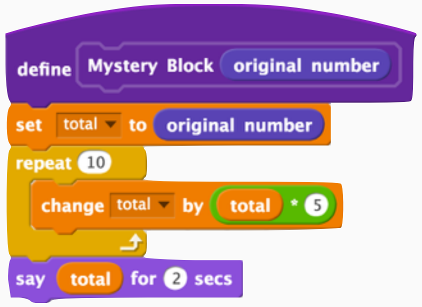
Contradiction

The Halting Problem

- *INPUT:*
    - A description of a program, input to the program.
- *OUTPUT:*
    - True if the program will halt when run on that input.
    - False if the program does not halt with that input.

The Halting Problem

- *INPUT:*



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  - A **description** of a program, **input** to the program.

- *OUTPUT:*
  - True if the program will halt when run on that input.
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The Halting Problem

- Suppose we have a correct algorithm for this problem:

The Halting Problem

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- Linear Search
- Dictionary
- “Snow”

The Halting Problem

- Suppose we have a correct algorithm for this problem:

The Halting Problem

- Suppose we have a correct algorithm for this problem:

- Random Search
- Dictionary
- “Flaboygle”

The Halting Problem

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Now consider the wily Gremlin:

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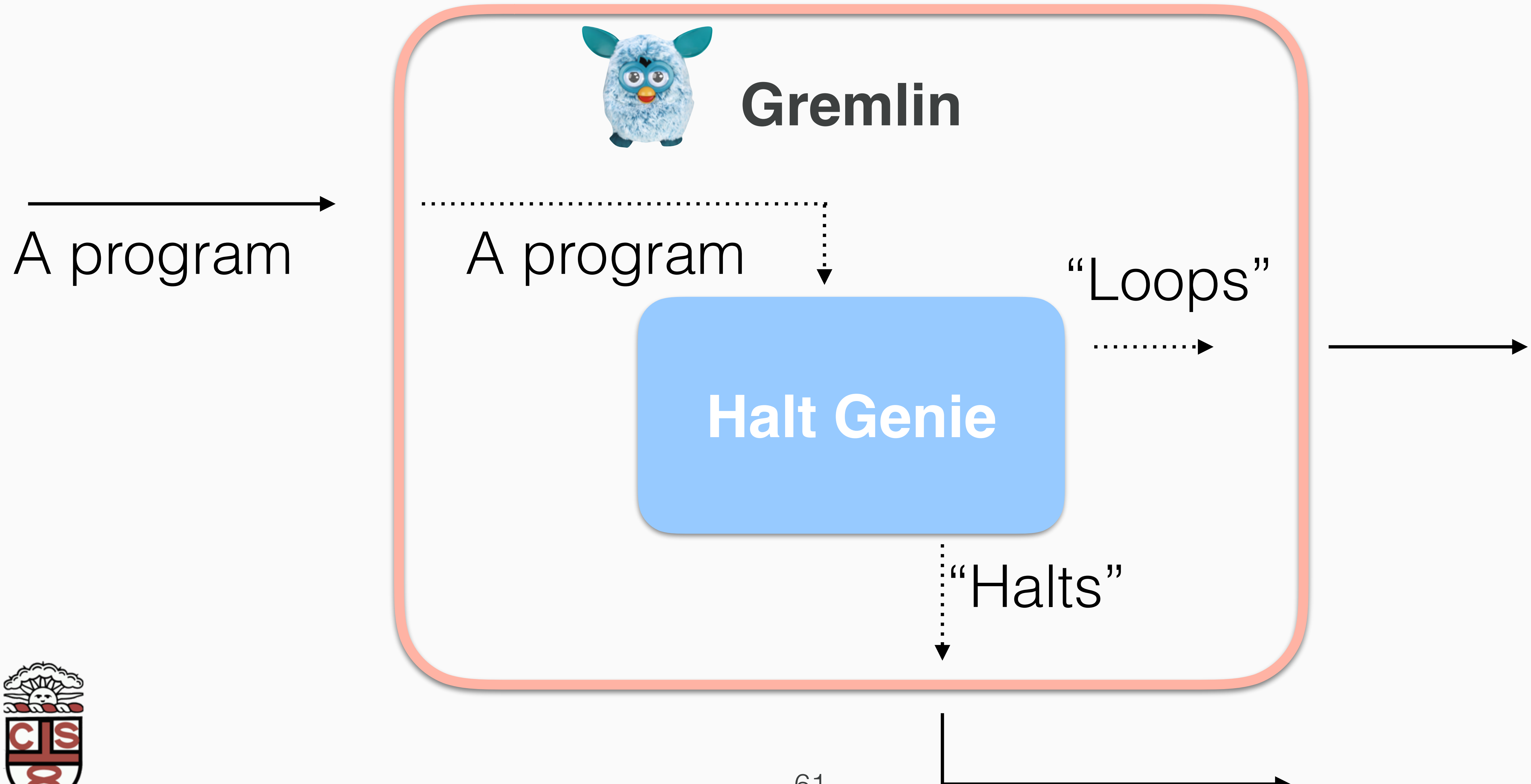

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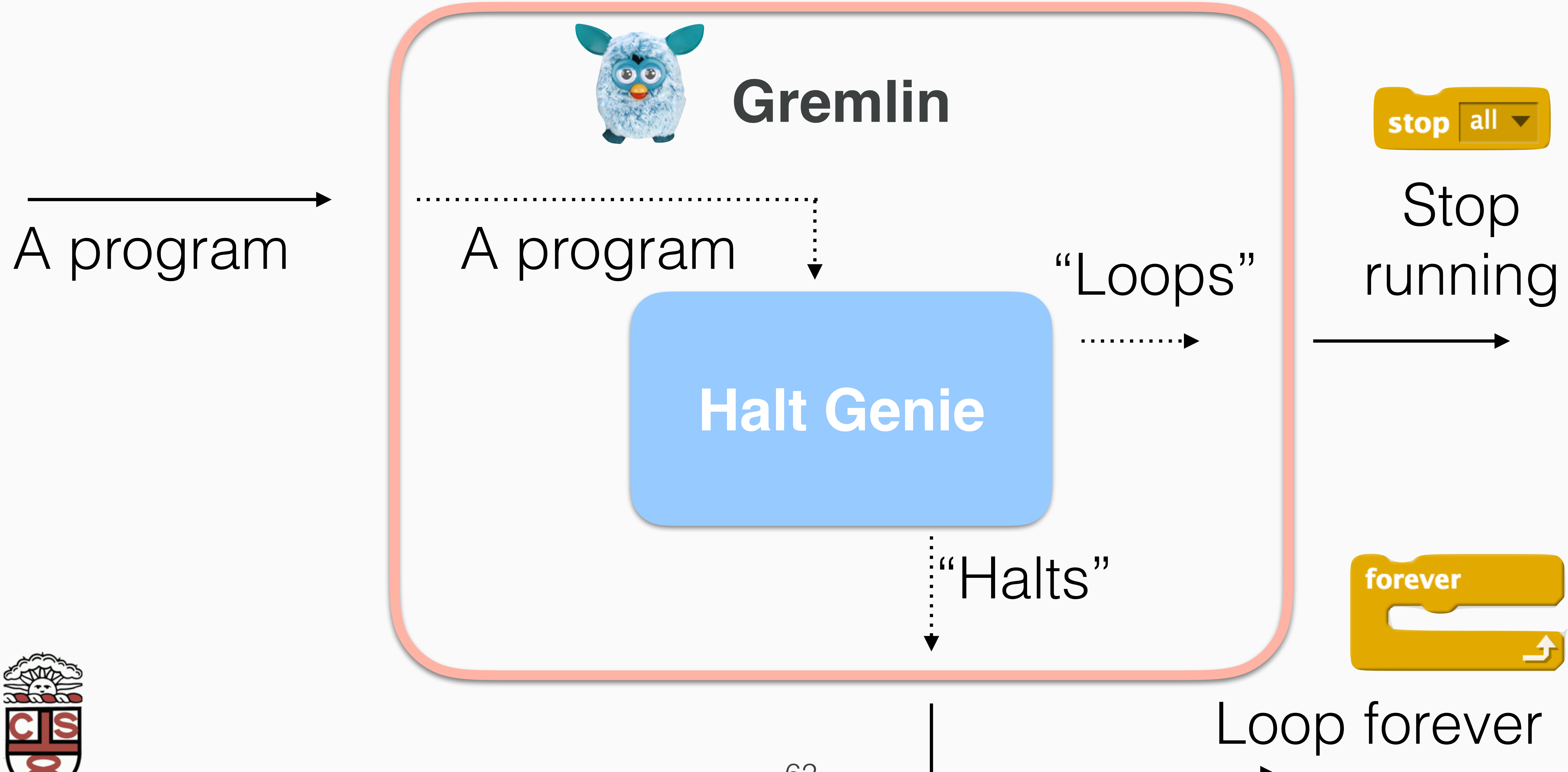

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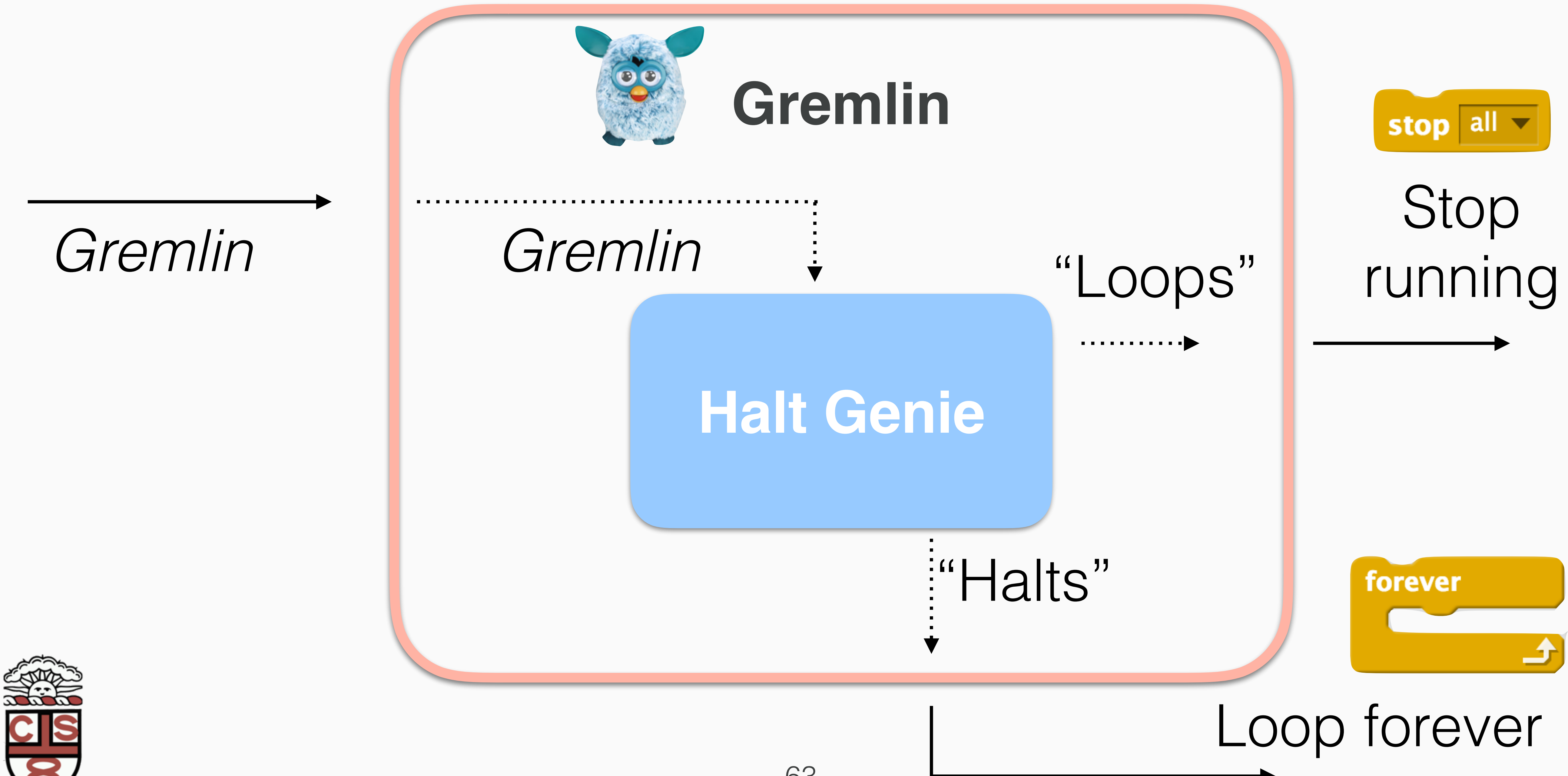

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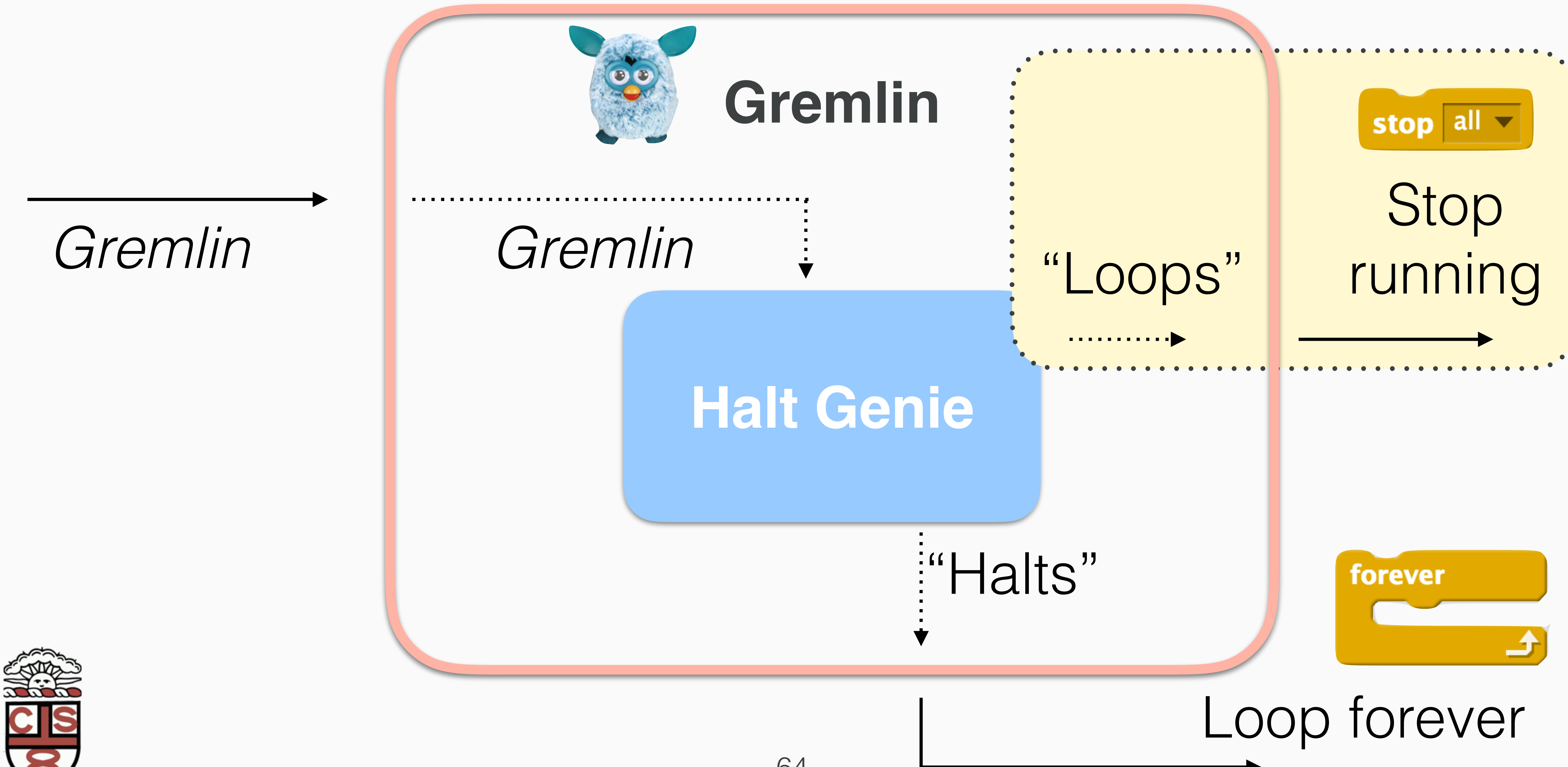

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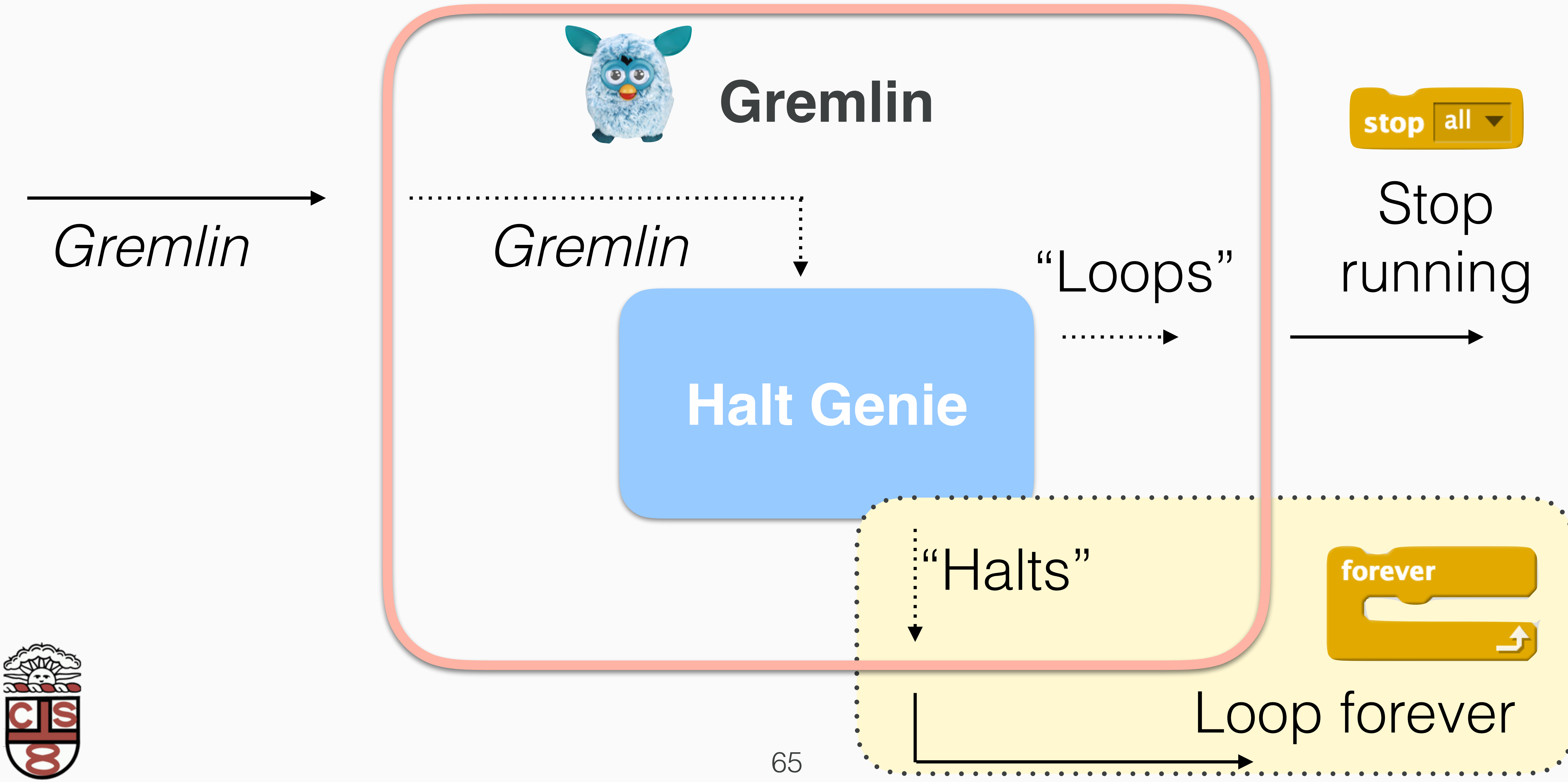

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If a Halt Genie exists (i.e. a correct algorithm for the Halting Problem), then we get a contradiction!

The Halting Problem

If a Halt Genie exists (i.e. a correct algorithm for the Halting Problem), then we get a contradiction!

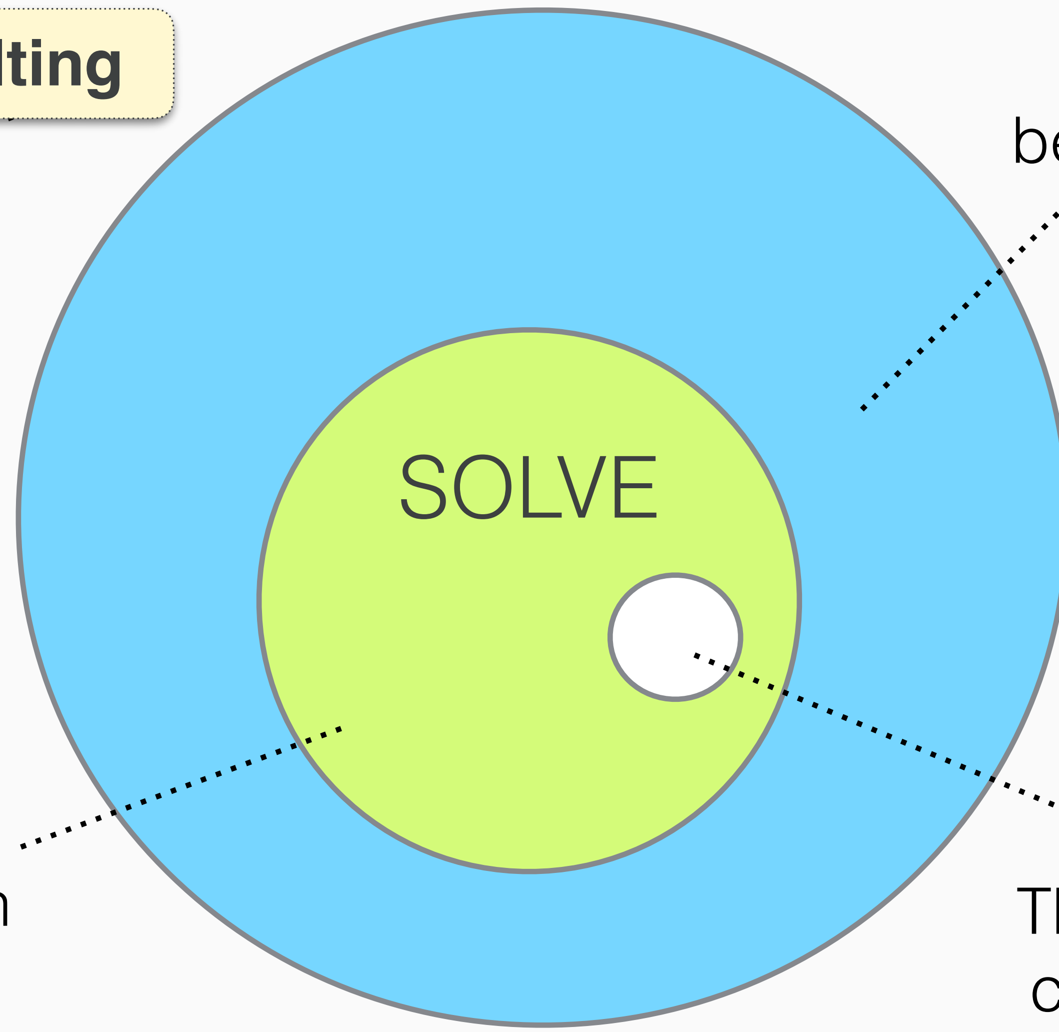
Therefore, no correct algorithm can exist for this problem.

Computability

Halting

Q: What, if anything, is out here?

A: The Halting Problem (and friends)



Things that can be computed, period.

Things a regular computer can compute before the sun goes supernova

Things a domino computer could compute before the sun goes supernova

Implications 1

- Closely related to Gödel's Incompleteness Theorems!
  - Mathematical statements that say of themselves, "I am not provable".
  - Result: there are mathematical statements that are unprovable, even though they're true.
  - Result: No mathematical system can prove certain properties about the system itself.
  - I.e. Can Logic prove that logic always makes the right conclusions?
- Gödel and Turing spoke at length about the relationship between consciousness, self-reference, and statements of this kind.
- Book: Gödel, Escher, Bach, an Eternal Golden Braid

Implications 2

- If we had a Halting Genie, we would have a pretty simple algorithm for solving any problem (even those that involve infinite loops).
  - Set it up so that, for *any question:*
    - "True" answers Halt
    - "False" answers Loop forever
  - Then, we just give the program to the Halting Genie. Boom! We have our answer.

Computable Numbers

- Consider:
  - 1,2,3,4,5….. ∞
- We could count twice as fast:
  - 2,4,6,8,10,… ∞
- Or even ten times as fast:
  - 10,20,30,40,… ∞

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    1-2-3-4-5
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    1-2-3-4

But with both of these, we can still count numbers in the sequence

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0.1

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Q: What about decimals?

0.1

0.01

Computable Numbers

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But with both of these, we can still count numbers in the sequence

Q: What about decimals?

0.1

0.01

0.001….

Computable Numbers

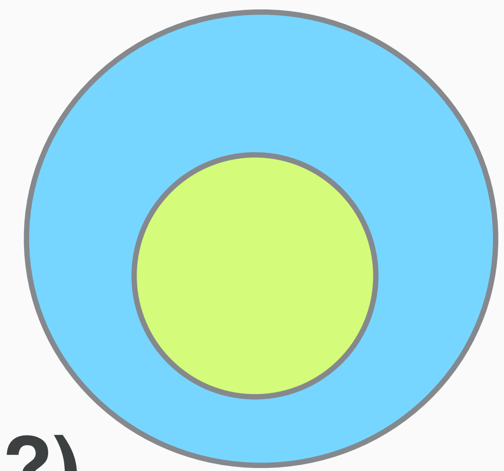
Things we can count	Things we can't count
1,2,3,4,5…	.01,.000001,.0000001…

Computable Numbers

Takeaway: there are numbers we cannot compute: those that are infinitely long decimals.

Reasoning: we can't even count them!

Reflection



- There are two notions of solvable:
  - **Computability (what's in the blue bubble?)**
    - What is solvable in principle (i.e. it doesn't matter how much time we have)
  - **Complexity (what's in SOLVE?):**
    - Given that we exist in *this* universe, and are subject to time constraints, how long do we have to sit around and wait for an answer?
- We're still trying to figure out which problems are in the second!